1. (10 pts.) The graph of a function $f(x)$ is shown. Using the same coordinate axis, sketch the graph of $y = f'(x)$.

![Graph of $y = f'(x)$]

2. (10 pts.) Find all points $(x, y)$ on the graph of $y = x^2 + \frac{16}{x^2}$ where the tangent line is horizontal.

$$y = x^2 + \frac{16}{x^2}$$

Slope $= \frac{dy}{dx} = 2x - \frac{32}{x^3}$

We seek $x$ that makes slope 0.

$$2x - \frac{32}{x^3} = 0$$

$$2x^4 - 32 = 0$$

$$x^4 = 16$$

$$x = \pm \sqrt[4]{16} = \pm 2$$

(These are the $x$ values that make slope = 0)

Point A

$(2, f(2)) = (2, 2^2 + \frac{16}{2^2})$

$= (2, 4 + 4)$

$= (2, 8)$

Point B

$(-2, f(-2)) = (-2, (-2)^2 + \frac{16}{(-2)^2})$

$= (-2, 4 + 4)$

$= (-2, 8)$

**Answer** $(2, 8)$ and $(-2, 8)$
3. (14 pts.) Find the indicated derivatives.

(a) \( f(\theta) = \sqrt{\theta^5} + \ln(\pi \theta) - \pi^2 = \theta^{5/2} + \ln(\pi \theta) - \pi^2 \)

\[ f'(\theta) = \frac{5}{2} \theta^{3/2} + \frac{\pi}{\pi \theta} - 0 = \frac{5}{2} \sqrt{\theta}^3 + \frac{1}{\theta} \]

\[ f''(\theta) = \frac{15}{4} \theta^{-1/2} - \frac{1}{\theta^2} = \frac{15}{4} \sqrt{\theta}^2 - \frac{1}{\theta^2} \]

(b) \( \frac{d}{dx} [(x^2 + x)\sqrt{3x+1}] = \frac{d}{dx} \left[ (x^2 + x) (3x+1)^{1/2} \right] = \)

\[ (2x+1) \sqrt{3x+1} + \frac{1}{2} (3x+1)^{-1/2} \cdot 3 = \]

\[ (2x+1) \sqrt{3x+1} + \frac{3(x^2+x)}{2\sqrt{3x+1}} \]

4. (21 pts.) Find the indicated derivatives.

(a) \( \frac{d}{dx} \left[ \frac{x^3+x^2+1}{x} \right] = \frac{(3x^2+2x)x - (x^3+x^2+1)(1)}{x^2} \)

(b) \( \frac{d}{dx} \left[ (\sec(\ln x))^3 \right] = 3 (\sec(\ln x))^2 \frac{d}{dx} [\sec(\ln x)] \)

\[ = 3 (\sec(\ln x))^2 \sec(\ln x) \tan(\ln x) \frac{1}{x} \]

(c) \( \frac{d}{dx} [\sec^{-1}(\pi x)] = \frac{\pi}{|\pi x| \sqrt{\pi x^2 - 1}} = \frac{1}{|x| \sqrt{(\pi x)^2 - 1}} \)
5. (10 pts.) Consider the equation \( x \tan(y^3) = y \). Use implicit differentiation to find \( \frac{dy}{dx} \).

Take \( \frac{d}{dx} \) of both sides:
\[
\frac{d}{dx} [x \tan(y^3)] = \frac{d}{dx} [y]
\]
\[
\frac{d}{dx} [x \tan(y^3)] + x \frac{d}{dx} [\tan(y^3)] = y'
\]

(1) \( \tan(y^3) + x \sec^2(y^3) \cdot 3y^2 \cdot y' = y' \)

Gather terms with \( y' \) on right-hand side:
\[
\tan(y^3) = y' - 3xy^2y' \sec^2(y^3)
\]

And factor out \( y' \):
\[
\tan(y^3) = y' (1 - 3xy^2 \sec^2(y^3)) \text{ and solve for } y'
\]

\[
\frac{\tan(y^3)}{1 - 3xy^2 \sec^2(y^3)} = y'
\]

6. (10 pts.) Use logarithmic differentiation to find the derivative of \( f(x) = x^{\sin(x)} \).

Let \( y = x^{\sin(x)} \). Take \( \ln \) of both sides:
\[
\ln(y) = \ln(x^{\sin(x)})
\]

Use log property \( \ln(x^r) = r \ln(x) \) for unlocking exponents:
\[
\ln(y) = \sin(x) \ln(x)
\]

Now it's an implicit diff. prob. Take \( \frac{d}{dx} \) of both sides:
\[
\frac{d}{dx} [\ln(y)] = \frac{d}{dx} [\sin(x) \ln(x)]
\]

\[
y' \frac{1}{y} = \sin(x) \frac{d}{dx} [\ln(x)] + \cos(x) \frac{d}{dx} [\sin(x)] \text{ product rule}
\]

\[
y' \frac{1}{y} = \cos(x) \ln(x) + \sin(x) \cdot \frac{1}{x}
\]

\[
y' = \cos(x) \ln(x) + \sin(x) \cdot \frac{1}{x}
\]

Plug in \( y = x^{\sin(x)} \):
\[
y' = (x^{\sin(x)})(\cos(x) \ln(x) + \sin(x) \cdot \frac{1}{x})
\]
7. (10 pts.) This problem concerns a rock that is thrown straight up in the air at time \( t = 0 \). At time \( t \) (in seconds) it has a height of \( s(t) = 64t - 16t^2 \) feet. Please show your work in answering the following questions.

(a) When does the rock hit the ground?

\[
\text{height} = 0 \quad \text{and} \quad t > 0 \quad \text{solve for } t \\
0 = 64t - 16t^2 = -16(t^2 - 4t) \quad \text{divide out } -16 \\
0 = t^2 - 4t = t(t-4) \quad \text{so } t = 0, 4 \text{ sec} \quad \text{so @ } t = 4 \text{ sec}
\]

(b) What is its velocity when it hits the ground?

\[
\begin{align*}
\text{find velocity } \nu(t) \text{ and evaluate at } t = 4 \\
\nu(4) &= 64 - 32t \\
\nu(4) &= 64 - 32(4) = -64 \text{ ft/sec}
\end{align*}
\]

8. (7 pts.) Simplify: \( \tan(\sin^{-1}(x)) = \)

\[
\tan(\sin^{-1}(x)) = \frac{\text{opp}}{\text{adj}} = \frac{x}{\sqrt{1-x^2}}
\]

9. (4 pts.)

(a) If \( f(x) = e^x \), then \( f^{-1}(x) = \ln x \).

(b) Carefully graph \( f(x) \) and \( f^{-1}(x) \) below.

10. (4 pts.)

(a) Graph the function \( g(x) = 1 - x^2 \) below.

(b) Now carefully graph the derivative \( g'(x) = -2x \).