Name: Richard

Sept. 14, 2012

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 1 ♢

(10 pts.) This problem concerns the functions $f(x) = \dfrac{\sqrt{x-1}}{5+\sin(x)}$ and $g(x) = \sqrt{x} - 1$.

(a) State the domain of $f(x)$.

The denominator is never zero (it's always positive!) so no problems there. But we must have $x - 1 \geq 0$ inside the radical, i.e. $x \geq 1$. Therefore $\boxed{\text{domain is } [1, \infty)}$

(b) $f \circ g(x) = f(g(x)) = \dfrac{\sqrt{g(x)-1}}{5+\sin(g(x))} = \dfrac{\sqrt{\sqrt{x}-1-1}}{5+\sin(\sqrt{x}-1)} = \dfrac{\sqrt{\sqrt{x}-2}}{5+\sin(\sqrt{x}-1)}$

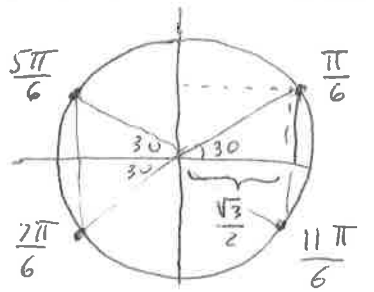
2. (10 pts.) Consider the equation $4\cos^2(x) - 3 = 0$. Find all solutions x that lie in the interval $[0, 2\pi)$.

$4\cos^2(x) = 3$

$\cos^2(x) = \frac{3}{4}$

$\cos(x) = \pm\sqrt{\frac{3}{4}}$

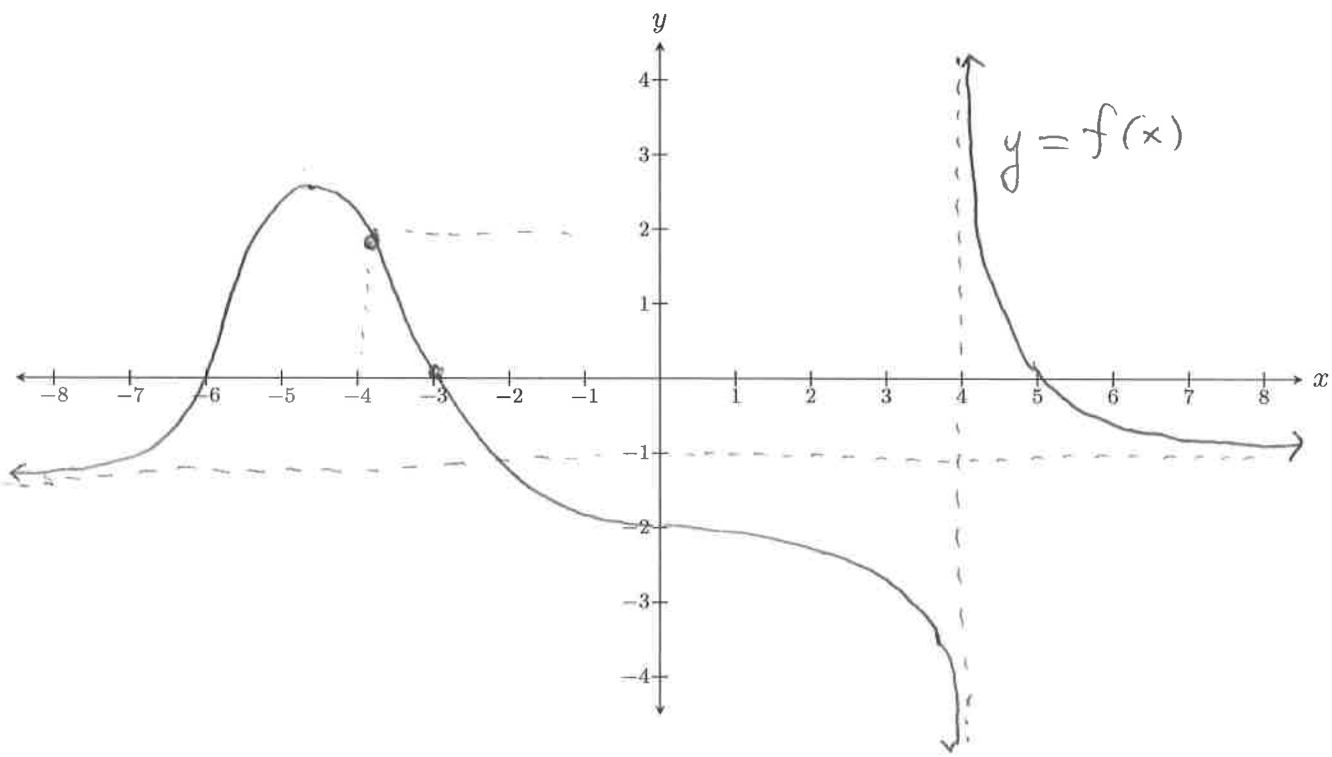
$\cos(x) = \pm\frac{\sqrt{3}}{2}$



By the unit circle, the values of x for which $\cos(x) = \pm\frac{\sqrt{3}}{2}$ are

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

(10 pts.) Sketch the graph of any function $y = f(x)$ that meets the following four criteria: The line $x = 4$ is a vertical asymptote, the line $y = -1$ is a horizontal asymptote, $f(-4) = 2$, and $\lim_{x \to -3} f(x) = 0$.



4. (20 pts.) Answer the following questions about the function $y = f(x)$ graphed below.

(a) $f(1) =$ $\boxed{-2}$

(b) $f \circ f(2) =$ $f(f(2)) = f(-1) = \boxed{-1}$

(c) $\lim\limits_{x\to 0} f(x) =$ $\boxed{0}$

(d) $\lim\limits_{x\to 1} f(x) =$ $\boxed{\text{DNE}}$

(e) $\lim\limits_{x\to 1^+} f(x) =$ $\boxed{-1}$

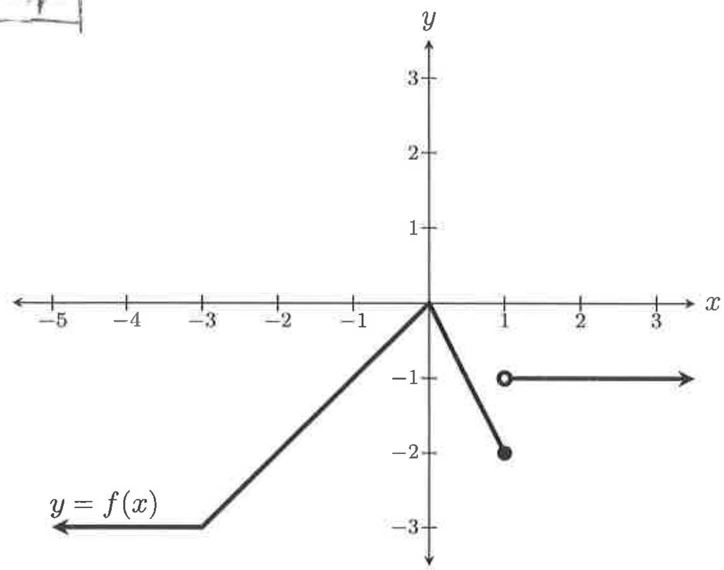
(f) $\lim\limits_{x\to 1^-} f(x) =$ $\boxed{-2}$

(g) $\lim\limits_{x\to \infty} f(x) =$ $\boxed{-1}$

(h) $\lim\limits_{x\to -\infty} f(x) =$ $\boxed{-3}$

(i) State an interval on which $f(x)$ is continuous. $\boxed{(1,\infty)}$

(j) State an x-value at which $f(x)$ is discontinuous. $\boxed{1}$



(28 pts.) Evaluate the following limits.
If you want credit, show your steps, explain your reasoning, and carry limits as appropriate.

(a) $\lim\limits_{x\to -1} \dfrac{x^2-3x-4}{x^2+5x+4} =$ $\lim\limits_{x\to 1} \dfrac{(x+1)(x-4)}{(x+1)(x+4)} = \lim\limits_{x\to -1} \dfrac{x-4}{x+4} = \dfrac{-1-4}{-1+4} = \dfrac{-5}{3} = \boxed{-\dfrac{5}{3}}$

(b) $\lim\limits_{h\to 0} \dfrac{\sqrt{5-h}-\sqrt{5}}{h} =$ $\lim\limits_{h\to 0} \dfrac{\sqrt{5-h}-\sqrt{5}}{h} \cdot \dfrac{\sqrt{5-h}+\sqrt{5}}{\sqrt{5-h}+\sqrt{5}} = \lim\limits_{h\to 0} \dfrac{5-h-5}{h(\sqrt{5-h}+\sqrt{5})}$

$= \lim\limits_{h\to 0} \dfrac{-h}{(\sqrt{5-h}+\sqrt{5})h} = \lim\limits_{h\to 0} \dfrac{-1}{\sqrt{5-h}+\sqrt{5}} = \dfrac{-1}{\sqrt{5-0}+\sqrt{5}} = \boxed{\dfrac{-1}{2\sqrt{5}}}$

(c) $\lim\limits_{x\to 3^-} \dfrac{(-x+3)(x+5)}{|-x+3|} =$ $\lim\limits_{x\to 3^-} \dfrac{(-x+3)}{|-x+3|}(x+5) = \lim\limits_{x\to 3^-} (1)(x+5) = 3+5 = \boxed{8}$

Note: $\dfrac{-x+3}{|-x+3|} = 1$ when x is to left of 3, e.g. $x=2$

(d) $\lim\limits_{\theta\to 0} \dfrac{\frac{1}{\theta}\sin(5\theta)}{\cos(\theta)} =$

$= \lim\limits_{\theta\to 0} \dfrac{\sin(5\theta)}{\theta} \dfrac{1}{\cos(\theta)} = 5 \lim\limits_{\theta\to 0} \dfrac{\sin(5\theta)}{5\theta} \dfrac{1}{\cos(\theta)} = 5\cdot(1)\left(\dfrac{1}{1}\right) = \boxed{5}$

Using $\dfrac{\sin(5\theta)}{5\theta} \to 1$ as $5\theta \to 0$

6. (12 pts.) Find all the horizontal asymptotes and vertical asymptotes of $f(x) = \frac{x^2+5x+4}{x^2+6x+8} = \frac{(x+1)(x+4)}{(x+2)(x+4)} = \frac{x+1}{x+2}$ provided $x \neq -2$

• To find the horizontal asymptotes we look at

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2+5x+4}{x^2+6x+8} \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1+\frac{5}{x}+\frac{4}{x^2}}{1+\frac{6}{x}+\frac{8}{x^2}} = \frac{1+0+0}{1+0+0} = 1$$

Similarly, $\lim_{x\to-\infty} f(x) = 1$ so $\boxed{\text{the line } y=1 \text{ is a H. A.}}$

• Candidates for vertical asymptotes are -2 and -4 because these make the dominator zero. Working the limits

$$\lim_{x\to-2^+} f(x) = \lim_{x\to-2^+} \frac{\overbrace{(x+1)}^{\text{close to } -1}}{\underbrace{(x+2)}_{\text{close to 0, neg}}} = \infty$$

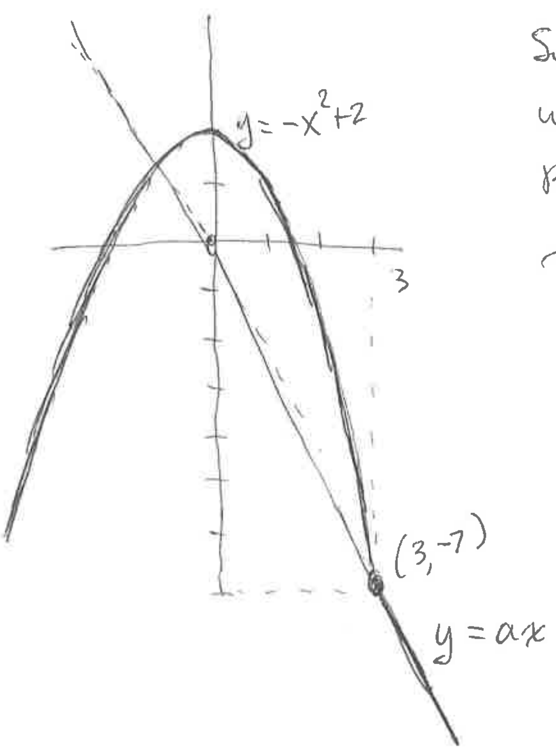
$\boxed{\text{line } \cancel{x} = -2 \text{ is V. A}}$

$$\lim_{x\to-4^+} f(x) = \lim_{x\to-4} \frac{x+1}{x+2} = \frac{-4+1}{-4+2} = \frac{-3}{2} = \frac{3}{2} \neq \infty$$

---

7. (10 pts.) Find the value a such that the following $f(x)$ is continuous at every number x.

$$f(x) = \begin{cases} -x^2+2 & \text{if } x < 3 \\ ax & \text{if } x \geq 3 \end{cases}$$



So that there is not a jump at $x=3$ we require that graph of $y=ax$ pass through $(3,-7)$ as indicated.

Then $y = ax$

$3 = a(-7)$

$$\boxed{a = \frac{-7}{3}}$$