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I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 1 ♣

1. (10 pts.) This problem concerns the functions $f(x) = \dfrac{\sqrt{x+1}}{\sin(x)+4}$ and $g(x) = \sqrt{x} - 1$.

(a) State the domain of $f(x)$. No value of x makes the denominator zero, so we don't have to worry about that. However, we must have $x+1 \geq 0$ inside the radical; Thus we require $x \geq -1$. Therefore $\boxed{\text{Domain} = [-1, \infty)}$

(b) $f \circ g(x) = f(g(x)) = \dfrac{\sqrt{g(x)+1}}{\sin(g(x))+4} = \dfrac{\sqrt{\sqrt{x}-1+1}}{\sin(\sqrt{x}+1)+4} = \dfrac{\sqrt{\sqrt{x}}}{\sin(\sqrt{x}+1)+4} = \dfrac{\sqrt[4]{x}}{\sin(\sqrt{x}+1)+4}$

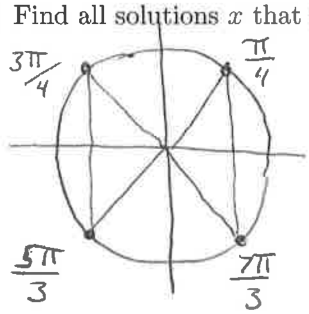
2. (10 pts.) Consider the equation $2\cos^2(x) - 1 = 0$. Find all solutions x that lie in the interval $[0, 2\pi)$.

$2\cos^2(x) - 1 = 0$

$2\cos^2(x) = 1$

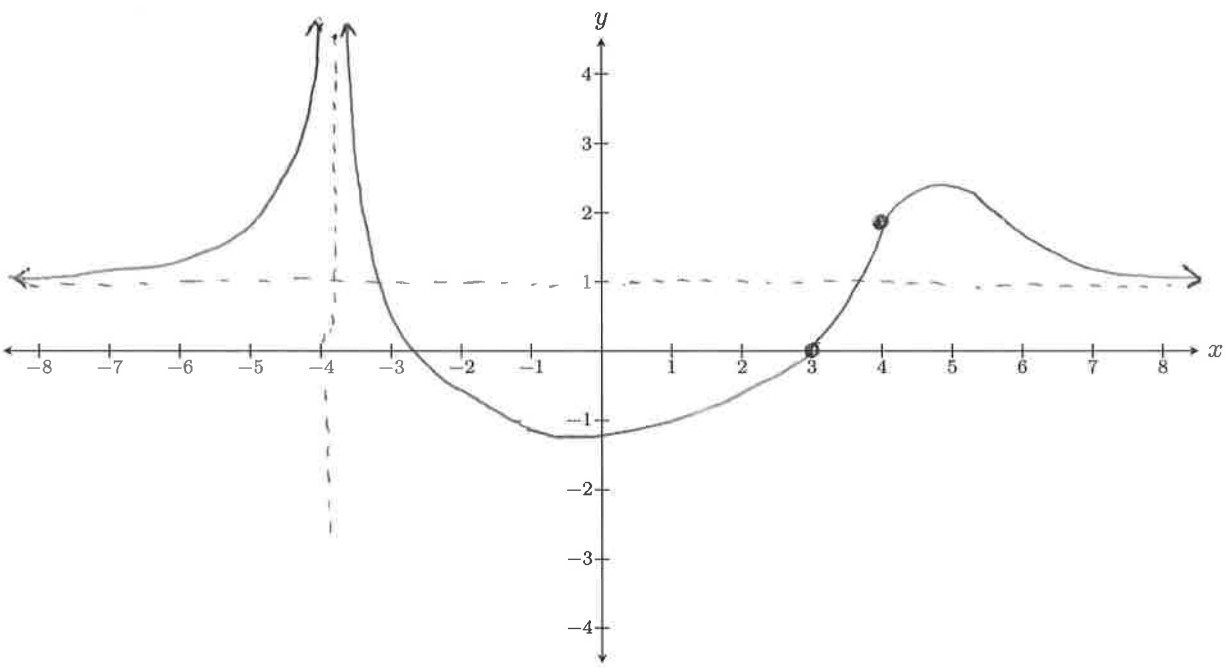
$\cos^2(x) = \frac{1}{2}$

$\boxed{\cos(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \frac{\pm\sqrt{2}}{2}}$



By the unit circle, the 4 values of $x \in [0, 2\pi)$ for which $\cos(x) = \frac{\pm\sqrt{2}}{2}$ are $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

3. (10 pts.) Sketch the graph of any function $y = f(x)$ that meets the following four criteria: The line $x = -4$ is a vertical asymptote, the line $y = 1$ is a horizontal asymptote, $f(4) = 2$, and $\lim_{x \to 3} f(x) = 0$.



4. (20 pts.) Answer the following questions about the function $y = f(x)$ graphed below.

(a) $f(1) = \boxed{-1}$

(b) $f \circ f(2) = f(f(2)) = f(0) = \boxed{1}$

(c) $\lim_{x \to 0} f(x) = \boxed{1}$

(d) $\lim_{x \to 1} f(x) = \boxed{\text{DNE}}$

(e) $\lim_{x \to 1^+} f(x) = \boxed{0}$

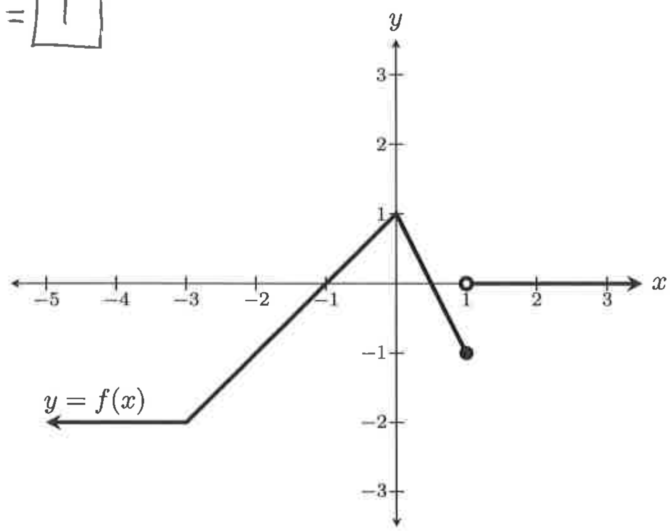
(f) $\lim_{x \to 1^-} f(x) = \boxed{-1}$

(g) $\lim_{x \to \infty} f(x) = \boxed{0}$

(h) $\lim_{x \to -\infty} f(x) = \boxed{-2}$

(i) State an interval on which $f(x)$ is continuous. $\boxed{(-\infty, 0]}$

(j) State an x-value at which $f(x)$ is discontinuous. $\boxed{x = 1}$



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5. (28 pts.) Evaluate the following limits.
If you want credit, show your steps, explain your reasoning, and carry limits as appropriate.

(a) $\lim_{x \to -2} \frac{x^2 - 4}{x^2 - x - 6} = \lim_{x \to -2} \frac{(x+2)(x-2)}{(x+2)(x-3)} = \lim_{x \to -2} \frac{x-2}{x-3} = \frac{-2-2}{-2-3} = \boxed{\frac{4}{5}}$

(b) $\lim_{h \to 0} \frac{\sqrt{7+h} - \sqrt{7}}{h} = \lim_{h \to 0} \frac{\sqrt{7+h} - \sqrt{7}}{h} \frac{\sqrt{7+h} + \sqrt{7}}{\sqrt{7+h} + \sqrt{7}} = \lim_{h \to 0} \frac{7 + h - 7}{h(\sqrt{7+h} + \sqrt{7})}$

$= \lim_{h \to 0} \frac{h}{h(\sqrt{7+h} + \sqrt{7})} = \lim_{h \to 0} \frac{1}{\sqrt{7+h} + \sqrt{7}} = \frac{1}{\sqrt{7+0} + \sqrt{7}} = \boxed{\frac{1}{2\sqrt{7}}}$

(c) $\lim_{x \to 3^+} \frac{(-x+3)(x+2)}{|-x+3|} = \lim_{x \to 3^+} \frac{(-x+3)}{|-x+3|}(x+2) = \lim_{x \to 3^+} (-1)(x+2) = -(3+2) = \boxed{-5}$

Note: $\frac{-x+3}{|-x+3|} = -1$ when x is to the right of 3

(d) $\lim_{\theta \to 0} \frac{\sec(\theta)}{\theta \csc(2\theta)} =$

$= \lim_{\theta \to 0} \frac{\frac{1}{\cos(\theta)}}{\theta \frac{1}{\sin(2\theta)}} = \lim_{\theta \to 0} \frac{\sin(2\theta)}{\theta} \frac{1}{\cos(\theta)} = \lim_{\theta \to 0} 2 \frac{\sin(2\theta)}{2\theta} \frac{1}{\cos(\theta)}$

$= 2 \cdot (1) \frac{1}{\cos(0)} = 2 \cdot 1 \cdot \frac{1}{1} = \boxed{2}$

6. (12 pts.) Find all the horizontal asymptotes and vertical asymptotes of $f(x) = \frac{x^2-2x-3}{x^2-1} = \frac{(x+1)(x-3)}{(x+1)(x-1)} = \frac{x-3}{x-1}$

To find the horozontal asymptotes, we examine the following limits:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2-2x-3}{x^2-1}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1-\frac{2}{x}-\frac{3}{x^2}}{1-\frac{1}{x^2}} = \frac{1-0-0}{1-0} = 1$$

Similarly, $\lim_{x\to-\infty} f(x) = 1$. Therefore $\boxed{\text{line } y=1 \text{ is a H.A.}}$

The candidates for vertical asymptotes are $x=-1$ and $x=1$, for these values make the denominator in $f(x)$ 0. Checking:

$$\left.\begin{aligned}\lim_{x\to-1^+} f(x) &= \lim_{x\to-1^+}\frac{x-3}{x-1} = \frac{-1-3}{-1-1} = \frac{-4}{-2} = 2 \neq \infty \\ \lim_{x\to1^+} f(x) &= \lim_{x\to1^+}\frac{\overbrace{(x-3)}^{\text{near } -2}}{\underbrace{(x-1)}_{\text{pos, near } 0}} = -\infty\end{aligned}\right\}$$

$\boxed{\text{Therefore the only vertical asymptote is the line } x=1}$

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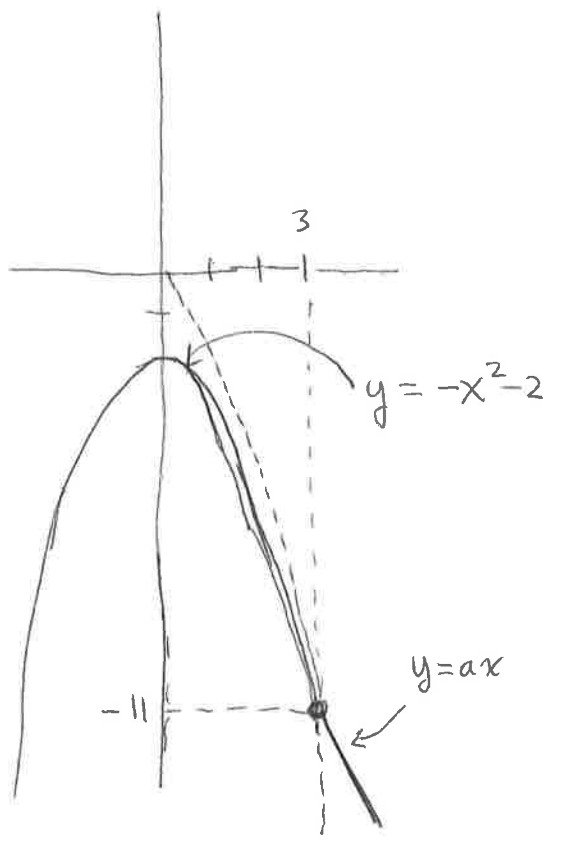
7. (10 pts.) Find the value a such that the following $f(x)$ is continuous at every number x.

$$f(x) = \begin{cases} -x^2-2 & \text{if } x < 3 \\ ax & \text{if } x \geq 3 \end{cases}$$



For the two parts to match up the point $(3,-11)$ should be on the line $y = ax$.

Thus $-11 = a\cdot 3$

Therefore $\boxed{a = -\frac{11}{3}}$