VCU
MATH 200
CALCULUS I
R. Hammack

TEST 1

May 23, 2014

Name: Richard

Score: 

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a box when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

(a) \[ \tan \left( \frac{5\pi}{3} \right) = \frac{\sin \left( \frac{5\pi}{3} \right)}{\cos \left( \frac{5\pi}{3} \right)} = \frac{-\sqrt{3}}{2} = \frac{-1}{2} = -\sqrt{3} \]

(b) Describe the domain of \( f(x) = \frac{x+1}{x\sqrt{x} + 5} \).

Require \( x+5 > 0 \) (i.e. \( x > -5 \)) and \( x \neq 0 \) so domain is \( (-5, 0) \cup (0, \infty) \).

(c) Suppose \( h(x) = \frac{\sin(\sqrt{x})}{\sqrt{x}} \).

State functions \( f(x) \) and \( g(x) \) for which \( h(x) = f \circ g(x) \).

\[ f(x) = \frac{\sin(x)}{x} \quad \text{and} \quad g(x) = \sqrt{x} \]

(d) \[
\lim_{x \to 3} \left( \frac{x^2 - 1}{x^3} \right)^{\frac{2}{3}} = \left( \lim_{x \to 3} \frac{x^2 - 1}{x^3} \right)^{\frac{2}{3}}
\]

\[
= \left( \frac{8}{27} \right)^{\frac{2}{3}} = \frac{8}{27} = \frac{4}{9} = \left( \frac{2}{3} \right)^2 = \left[ \frac{4}{9} \right]
\]

(e) \[
\lim_{x \to \frac{\pi}{2}^+} \tan(x) = -\infty
\]
2. (15 points) Consider the equation \(2\sin^2(x) = -\sin(x)\).

Find all solutions \(x\) of this equation for which \(0 \leq x \leq 2\pi\).

\[
2\sin^2(x) = -\sin(x)
\]

\[
2\left(\sin(x)\right)^2 + \sin(x) = 0
\]

\[
\sin(x)\left(2\sin(x) + 1\right) = 0
\]

\[
\sin(x) = 0 \quad \text{or} \quad 2\sin(x) + 1 = 0
\]

\[
2\sin(x) = -1 \quad \sin(x) = -\frac{1}{2}
\]

\[
\sin(x) = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}
\]

Answer: solutions are

\[x = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}\]
3. (15 points) Evaluate the following limits.

(a) \[ \lim_{x \to 2} \frac{\sin(2x - 4)}{5x - 10} = \lim_{x \to 2} \frac{1}{5} \frac{\sin(2x - 4)}{x - 2} \]

\[ \quad = \lim_{x \to 2} \frac{2 \sin(2x - 4)}{5 \cdot (x - 2)} \]

\[ \quad = \lim_{x \to 2} \frac{2}{5} \frac{\sin(2x - 4)}{2x - 4} = \frac{2}{5}, 1 = \frac{2}{5} \]

(b) \[ \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} = \lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h} \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} \]

\[ \quad = \lim_{h \to 0} \frac{4+h - 2}{h} \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} \]

\[ \quad = \lim_{h \to 0} \frac{h}{h} \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} = \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{2} \]

(c) \[ \lim_{x \to 3} \frac{1}{x^2 - 9} = \lim_{x \to 3} \frac{\frac{1}{x^2 - 9}}{x - 3} \]

\[ \quad = \lim_{x \to 3} \frac{\frac{9 - x^2}{9x^2}}{x^2 - 9} \]

\[ \quad = \lim_{x \to 3} \frac{9 - x^2}{9x^2} \frac{1}{x - 3} \]

\[ \quad = \lim_{x \to 3} \frac{9 - x^2}{9x^2} \frac{1}{x - 3} = \lim_{x \to 3} \frac{(3-x)(3+x)}{9x^2(x - 3)} \]

\[ \quad = \lim_{x \to 3} \frac{-(3+x)}{9x^2} = -\frac{3 + 3}{9 \cdot 3^2} = -\frac{6}{81} = -\frac{2}{27} \]
4. (15 points) Sketch the graph of any function that meets all of the following criteria.

1. \( f(-1) = 3 \)

2. \( \lim_{x \to \infty} f(x) = -2 \)

3. The line \( y = 3 \) is a horizontal asymptote

4. \( \lim_{x \to 2^+} f(x) = -\infty \) and \( \lim_{x \to 2^-} f(x) = \infty \)

5. \( \lim_{x \to -1} f(x) = 2 \)

6. \( f(x) \) continuous at every \( x \) value except \( x = -1 \) and \( x = 2 \)
5. (15 points) This question concerns the function \( f(x) = \frac{15 - 12x - 3x^2}{50 - 2x^2} \).

(a) State the intervals on which \( f(x) \) is continuous.

\[
f(x) = \frac{-3(5 + 4x + x^2)}{2(25 - x^2)} = \frac{-3(x^2 + 4x - 5)}{2(5-x)(5+x)} = \frac{-3(x-1)(x+5)}{2(5-x)(5+x)}
\]

Not continuous at \( x = 5, -5 \)

Continuous on \([-\infty, -5) \cup (-5, 5) \cup (5, \infty)\)

(b) Find the horizontal asymptotes (if any).

From looking at coefficients of highest powers, \( \lim_{x \to \infty} f(x) = \frac{-3}{-2} = \frac{3}{2} \)

Thus \( y = \frac{3}{2} \) is a H.A.

(c) Find the vertical asymptotes (if any).

These could be located at either \( x = 5 \) or \( x = -5 \), where the denominator of \( f(x) \) is zero.

Note by factoring above \( f(x) = \frac{-3(x-1)}{2(5-x)} \)

Test \( x = 5 \): \( \lim_{x \to 5^+} \frac{-3(x-1)}{2(5-x)} = \infty \)

Thus \( \text{line } x = 5 \text{ is a VA} \)

Test \( x = -5 \): \( \lim_{x \to -5^+} \frac{-3(x-1)}{2(5-x)} = \frac{-3(-5-1)}{2(5-(-5))} = \frac{18}{20} \neq \pm \infty \) so no vertical asymptote here.
6. (15 points) Two functions $f(x)$ and $g(x)$ are graphed below. Answer the following questions.

(a) $\lim_{x \to 3} f(x) = 2$

(b) Find $c$ if $\lim_{x \to c} f(x) = 0$. $c = -1$

(c) $\lim_{x \to -2} \frac{3f(x)g(x)}{\sqrt{12 + f(x)}} = \frac{3(-3)(1)}{\sqrt{12 + (-3)}} = \frac{-9}{\sqrt{9}} = -3$

(d) $g \circ f(-2) = g(f(-2)) = g(-3) = \frac{1}{2}$

(e) $\lim_{x \to 3} f(g(x)) = f(\lim_{x \to 3} g(x)) = f(2) = 1$