Name: __________________________

Score: __________

Directions. Solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not used. Put a your final answer in a box where appropriate.

1. (20 points) Warmup: short answer.

(a) \( \frac{d}{dx} \left[ \sin(x) + \cos(x) \right] = \)

(b) \( \frac{d}{dx} \left[ \sin(x) \cos(x) \right] = \)

(c) \( \frac{d}{dx} \left[ \sin(\cos(x)) \right] = \)

(d) \( \frac{d}{dx} \left[ e^x \right] = \)

(e) \( \frac{d}{dx} \left[ x^e \right] = \)

(f) \( \ln(1/e) = \)

(g) \( \sin^{-1}(1/2) = \)

(h) \( e^{\cos(\pi/2)} = \)

(i) \( \lim_{x \to -\infty} e^x = \)

(j) \( \lim_{x \to \infty} \tan^{-1}(x) = \)

7. (10 points) Sketch the graph of both \( y = \ln(x) \) and \( y = e^x \) below. Be sure to indicate which graph is which.

8. (5 points) \( \sin(\tan^{-1}(x)) = \)
2. (10 points) Answer the following questions concerning the function \( f(x) \) graphed below.

(a) Using the coordinate axis above, sketch the graph of the derivative \( y = f'(x) \).

(b) Suppose \( g(x) = (f(x))^3 \). Find \( g'(-2) \).

3. (15 points) An object moving on a straight line is \( s(t) = 2 + t + t^3 \) feet from its starting point at time \( t \) seconds.

(a) What is the object’s velocity at time \( t \)?

(b) What is its acceleration at time \( t \)?

(c) Find its velocity when its acceleration is 12 feet per second per second.

4. (10 points) This problem concerns the functions \( f(x) = x^3 - 3x \) and \( g(x) = 3x^2 + 6x \). Find all \( x \) for which the tangent to \( y = f(x) \) at \( (x, f(x)) \) is parallel to the tangent to \( y = g(x) \) at \( (x, g(x)) \).
5. (20 points) Find the following derivatives.

(a) \( \frac{d}{dx} \left[ \ln(x) + \frac{1}{x} + \sqrt{x} + 3 \right] = \)

(b) \( \frac{d}{dx} \left[ \left( \frac{x^2 + 5}{x + 1} \right)^4 \right] = \)

(c) \( \frac{d}{dx} \left[ \tan^{-1}(5x) \right] = \)

(d) \( \frac{d}{dx} \left[ x \sec(e^{10x}) \right] = \)

6. (10 points) This question concerns the equation \( x^2 + xy + y^2 = 7. \)

(a) Use implicit differentiation to find \( \frac{dy}{dx}. \)

(b) Use your answer from part (a) to find the equation of the tangent line to the graph of \( x^2 + xy + y^2 = 7 \) at the point \((2, -3).\)