Name: Richard

QUIZ 3

MATH 200
August 31, 2023

1. $\lim_{x \to \frac{\pi}{4}} \log_2 (2\sin(x)) = \log_2 \left(\lim_{x \to \frac{\pi}{4}} 2\sin(x) \right) = \log_2 \left(2 \lim_{x \to \frac{\pi}{4}} \sin(x) \right)$

$= \log_2 \left(2 \sin\left(\frac{\pi}{4}\right) \right) = \log_2 \left(2 \frac{\sqrt{2}}{2} \right) = \log_2(\sqrt{2}) = \boxed{\frac{1}{2}}$

2. $\lim_{x \to 1} \frac{\sin(x-1)}{2-2x} = \lim_{x \to 1} \frac{\sin(x-1)}{-2(x-1)} = -\frac{1}{2} \lim_{x \to 1} \frac{\sin(x-1)}{(x-1)} = \boxed{-\frac{1}{2}}$

approaching 0

3. $\lim_{x \to 0} \frac{3 - 3\cos(x)}{\cos(x) - 1} = \lim_{x \to 0} \frac{-3\cancel{(\cos(x)-1)}}{\cancel{\cos(x)-1}} = \lim_{x \to 0}(-3) = \boxed{-3}$

4. This problem concerns the function $f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2 \\ x^2 - cx, & \text{if } x \geq 2 \end{cases}$

Find the value(s) of c such that f will be continuous at all x. Show and explain your work.

Because cx^2+2x and x^2-cx are polynomials, this function is automatically continuous on $(-\infty, 2) \cup (2, \infty)$. Thus we must find the value of c that makes $f(x)$ continuous at $x=2$. Notice that:

$f(2) = 2^2 - c \cdot 2 = \boxed{4-2c}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 - cx) = 2^2 - c \cdot 2 = \boxed{4-2c}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (cx^2 + 2x) = c \cdot 2^2 + 2 \cdot 2 = \boxed{4c+4}$

For continuity at $x=2$ the right- and left-hand limits must both equal $f(2) = 4-2c$ that is,

$4c + 4 = 4 - 2c \implies 6c = 0 \implies$ $\boxed{\text{Answer } c = 0}$

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1. $\lim_{x\to\frac{\pi}{6}} \log_2(\sin(x)) = \log_2\left(\lim_{x\to\frac{\pi}{6}} \sin(x)\right) = \log_2\left(\sin\left(\frac{\pi}{6}\right)\right)$

$= \log_2\left(\frac{1}{2}\right) = \boxed{-1}$

2. $\lim_{x\to\frac{\pi}{2}} \frac{\sin(x)-1}{2-2\sin(x)} = \lim_{x\to\frac{\pi}{2}} \frac{\sin(x)-1}{-2(-1+\sin(x))} = -\frac{1}{2}\lim_{x\to\frac{\pi}{2}} \frac{\sin(x)-1}{\sin(x)-1}$

getting $\frac{0}{0}$

$= -\frac{1}{2}\lim_{x\to\frac{\pi}{2}} 1 = -\frac{1}{2}\cdot 1 = \boxed{-\frac{1}{2}}$

3. $\lim_{x\to 1} \frac{\sin(x-1)}{2-2x} = \lim_{x\to 1} \frac{\sin(x-1)}{-2(-1+x)} = -\frac{1}{2}\lim_{x\to 1}\frac{\sin(x-1)}{x-1} = -\frac{1}{2}\cdot 1 = \boxed{-\frac{1}{2}}$

getting $\frac{0}{0}$

approaching 0

4. This problem concerns the function $f(x) = \begin{cases} cx^2+2x, & \text{if } x<2 \\ x^2-cx, & \text{if } x\geq 2 \end{cases}$

Find the value(s) of c such that f will be continuous at all x. Show and explain your work.

Because cx^2+2x and x^2-cx are polynomials, this function is automatically continuous on $(-\infty,2)\cup(2,\infty)$. Thus we must find the value of c that makes $f(x)$ continuous at $x=2$. Notice that:

$f(2) = 2^2 - c\cdot 2 = \boxed{4-2c}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^2-cx) = 2^2-c\cdot 2 = \boxed{4-2c}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(cx^2+2x) = c\cdot 2^2+2\cdot 2 = \boxed{4c+4}$

For continuity at $x=2$, the right- and left-hand limits must both equal $f(2)$, that is,

$4-2c = 4c+4 \implies 6c=0 \implies$ **Answer: $c=0$**