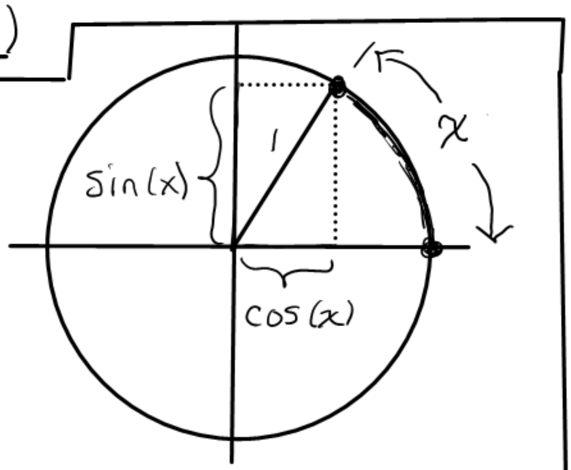
Review of Trig Functions (Continued)

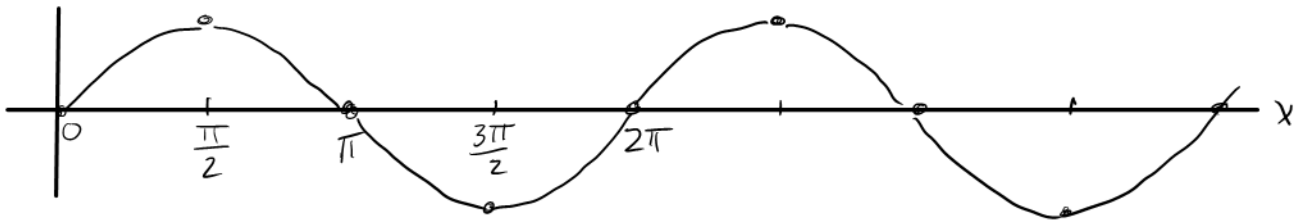
Recall how sin and cos are defined from the unit circle



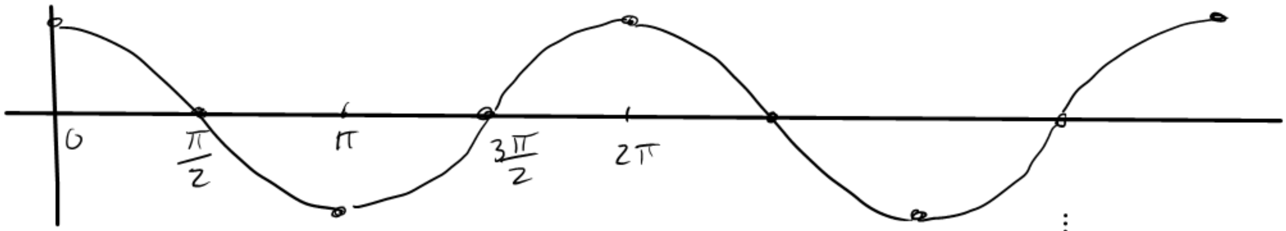
Also: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ $\quad \sec(x) = \dfrac{1}{\cos(x)}$

$\cot(x) = \dfrac{\cos(x)}{\sin(x)}$ $\quad \csc(x) = \dfrac{1}{\sin(x)}$

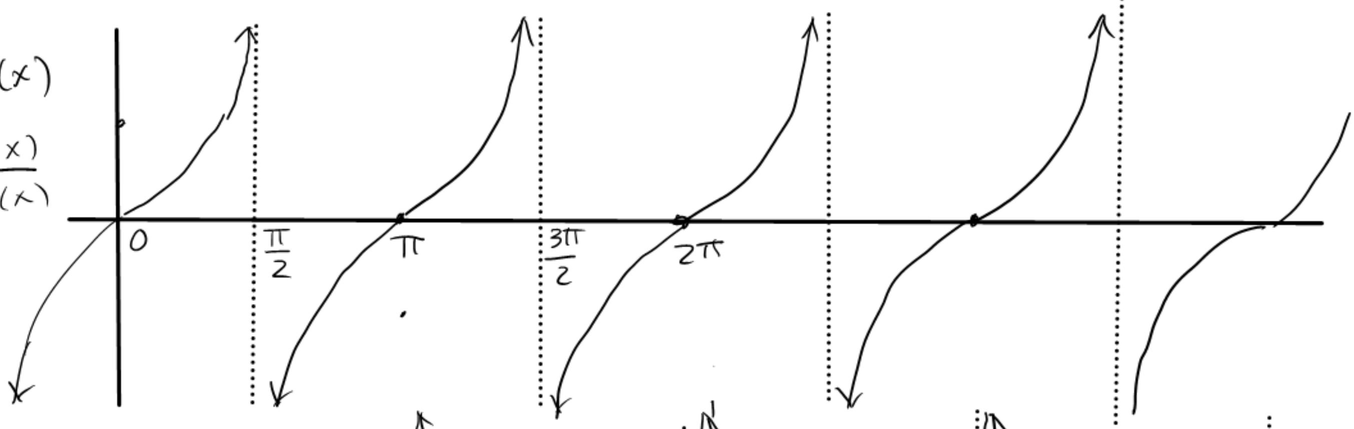
$y = \sin(x)$



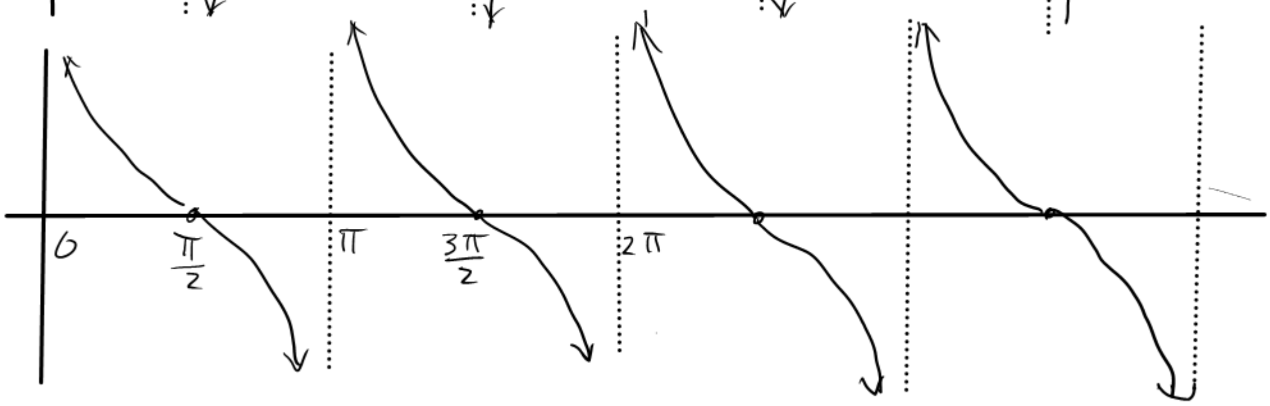
$y = \cos(x)$



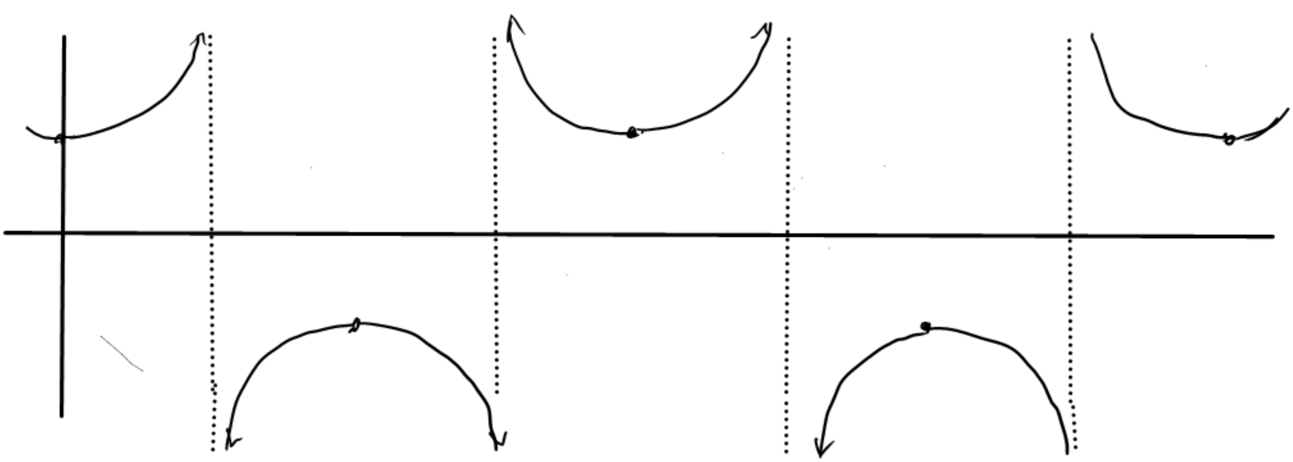
$y = \tan(x)$
$= \dfrac{\sin(x)}{\cos(x)}$



$y = \cot(x)$
$= \dfrac{\cos(x)}{\sin(x)}$

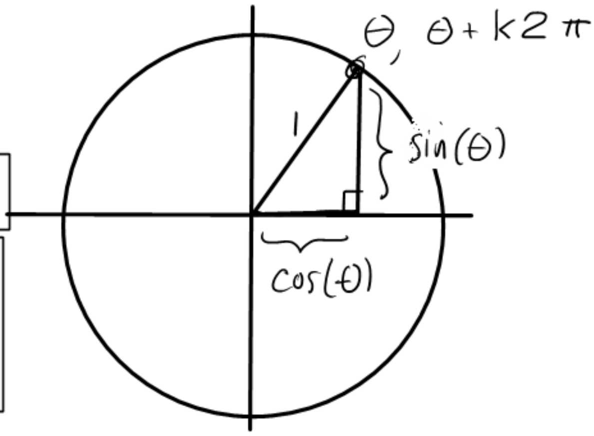


$y = \sec(x)$
$= \dfrac{1}{\cos(x)}$



Trigonometric Identities (True for any value of θ)

By Pythagorean Theorem:



$$\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$$

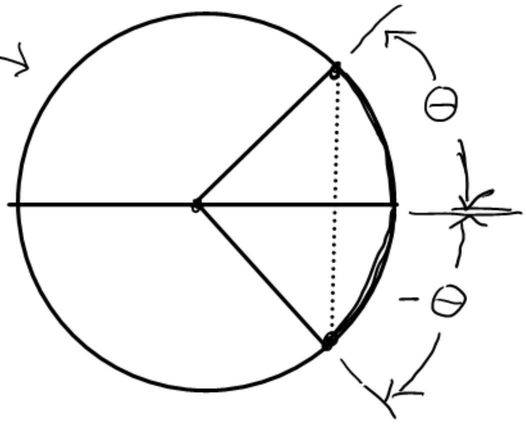
Also

$$\left.\begin{aligned} \sin(\theta + 2k\pi) &= \sin(\theta) \\ \cos(\theta + 2k\pi) &= \cos(\theta) \end{aligned}\right\} k = 0, \pm 1, \pm 2 \ldots$$

And

$$\cos(-\theta) = \cos(\theta)$$
$$\sin(-\theta) = -\sin(\theta)$$

from picture



More identities:

Addition Formulas

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

Double Angle Formulas

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$
$$\sin(2\alpha) = 2\cos(\alpha)\sin(\alpha)$$

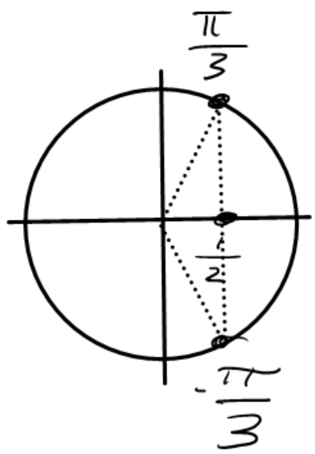
Half Angle Formulas

$$\cos^2(\alpha) = \frac{1}{2}\left(1 + \cos(2\alpha)\right)$$
$$\sin^2(\alpha) = \frac{1}{2}\left(1 - \cos(2\alpha)\right)$$

<u>Example</u> $\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1}{2}\left(1 + \cos\left(2 \cdot \frac{\pi}{8}\right)\right)} = \sqrt{\frac{1}{2}\left(1 + \cos\frac{\pi}{4}\right)} = \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{2+\sqrt{2}}}{2}$

Solving Trig Equations

<u>Example</u> Find all solutions of $\cos(x) = \frac{1}{2}$



From the unit circle we can see that the set of all solutions is

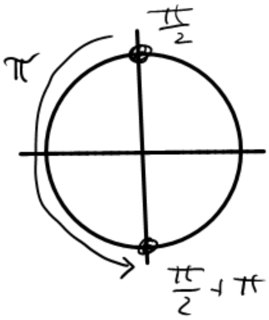
$$\left.\begin{aligned} x &= \frac{\pi}{3} + 2k\pi \\ x &= -\frac{\pi}{3} + 2k\pi \end{aligned}\right\} k = 0, \pm 1, \pm 2, \pm 3 \ldots$$

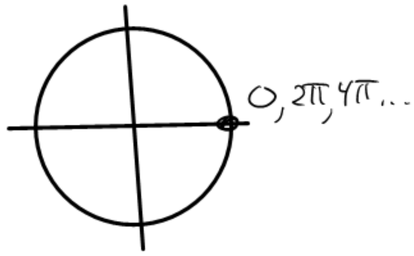
Example Solve $\cos^2(x) - \cos(x) = 0$

$$\cos(x)(\cos(x) - 1) = 0$$

$\cos(x) = 0$ $\qquad\qquad$ $\cos(x) - 1 = 0$

$\qquad\qquad\qquad\qquad$ $\cos(x) = 1$



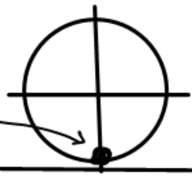


$$\boxed{\text{Answer: } \left.\begin{aligned} x &= \tfrac{\pi}{2} + k\pi \\ x &= 2k\pi \end{aligned}\right\} k = 0, \pm 1, \pm 2, \pm 3, \ldots}$$

Example Find the domain of $f(x) = \dfrac{1}{1 + \sin(x)}$

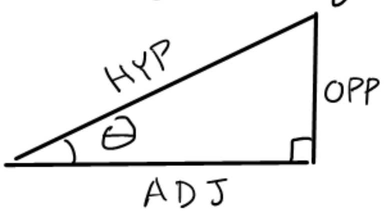
Domain is all x except those for which $1 + \sin(x) = 0$, that is $\sin(x) = -1$

$x = \frac{3\pi}{2}$



Answer All real numbers **except** $x = \frac{3\pi}{2} + 2k\pi$ for $k = 0, \pm 1, \pm 2, \pm 3, \ldots$

Solving Triangles (finding lengths of missing sides, etc.

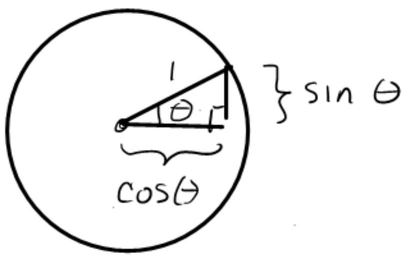


By similar triangles:

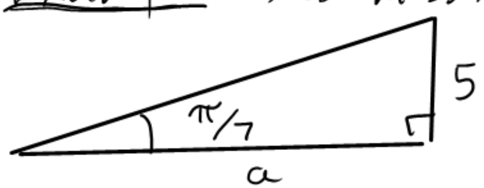
$\dfrac{\sin\theta}{1} = \dfrac{OPP}{HYP}$ $\longrightarrow$ $\boxed{\sin\theta = \dfrac{OPP}{HYP}}$

$\dfrac{\cos\theta}{1} = \dfrac{ADJ}{HYP}$ $\longrightarrow$ $\boxed{\cos\theta = \dfrac{ADJ}{HYP}}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{OPP}{ADJ}$ $\longrightarrow$ $\boxed{\tan\theta = \dfrac{OPP}{ADJ}}$



Example Find missing side a



$$\frac{5}{a} = \frac{OPP}{ADJ} = \tan\left(\frac{\pi}{7}\right)$$

$$a = \frac{5}{\tan\left(\frac{\pi}{7}\right)} \approx \boxed{10.3826}$$