Limit Practice

Recall the following ideas / facts:

- In evaluating $\lim_{x \to c} \frac{f(x)}{g(x)}$, if $g(c) = 0$, then try to cancel the term that makes $g(c) = 0$.
- $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$
- $\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$

Examples

- $\lim_{x \to -3} \frac{x+3}{x^2+4x+3} = \lim_{x \to -3} \frac{\cancel{x+3}}{\cancel{(x+3)}(x+1)} = \lim_{x \to -3} \frac{1}{x+1} = \frac{1}{-3+1} = \boxed{-\frac{1}{2}}$

Remark:

Never do this:

$$\lim_{x \to -3} \frac{x+3}{(x+3)(x+1)} = \frac{1}{x+1} = \frac{1}{-3+1}$$

not equal!! not equal!!

Continue writing $\lim_{x \to c}$ until you get to a point where you can actually plug in $x = c$ or otherwise evaluate the limit

Points may be deducted wherever you assert two things are equal when they are not actually equal!!

Likewise, don't write your final step as, say, $\boxed{\lim_{x \to -3} = -\frac{1}{2}}$

Syntax is $\lim_{x \to -3} f(x) = -\frac{1}{2}$

- $$\lim_{h\to 0} \frac{\sqrt{5+h}-\sqrt{5}}{h} = \lim_{h\to 0} \frac{\sqrt{5+h}-\sqrt{5}}{h} \frac{\sqrt{5+h}+\sqrt{5}}{\sqrt{5+h}+\sqrt{5}}$$

$$= \lim_{h\to 0} \frac{\sqrt{5+h}^2-\sqrt{5}^2}{h(\sqrt{5+h}+\sqrt{5})} = \lim_{h\to 0} \frac{5+h-5}{h(\sqrt{5+h}+\sqrt{5})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{5+h}+\sqrt{5})} = \lim_{h\to 0} \frac{1}{\sqrt{5+h}+\sqrt{5}} = \frac{1}{\sqrt{5+0}+\sqrt{5}} = \boxed{\frac{1}{2\sqrt{5}}}$$

- $$\lim_{x\to 0} \frac{\sin(2x)}{5x} = \frac{1}{5} \lim_{x\to 0} \frac{\sin(2x)}{x} = \frac{2}{5} \lim_{x\to 0} \frac{\sin(2x)}{2x}$$

$$\lim_{\{\}\to 0} \frac{\sin(\{\})}{\{\}} = 1$$

$$= \frac{2}{5}(1) = \boxed{\frac{2}{5}}$$

- $$\lim_{x\to -2} \frac{\sin(x^2+3x+2)}{x+2} = \lim_{x\to -2} \frac{\sin(x^2+3x+2)}{x^2+3x+2} \frac{x^2+3x+2}{x+2}$$

$(x^2+3x+2) \to 0$ as $x \to 0$

$$= \lim_{x\to -2} \frac{\sin(x^2+3x+2)}{x^2+3x+2} \frac{(x+2)(x+1)}{x+2}$$

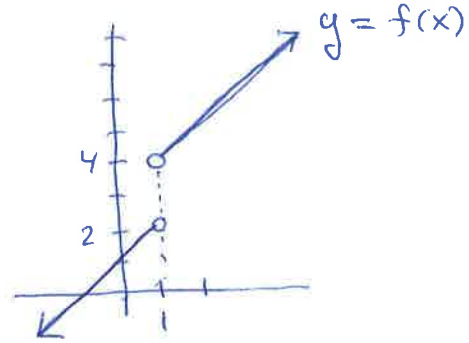
$$= \lim_{x\to -2} \frac{\sin(x^2+3x+2)}{x^2+3x+2}(x+1) = (1)(-1) = \boxed{-1}$$

Section 2.4 One-Sided Limits (Continued)

Recall that to have $\lim_{x \to c} f(x) = L$ it must be the case that $f(x)$ approaches L no matter how x "gets to" the number c. About a week ago we worked the following example.

Ex $f(x) = x + \dfrac{|x-1|}{x-1} + 2$

$$= \begin{cases} x+3 & \text{if } x<1 \\ x+1 & \text{if } x>1 \end{cases}$$



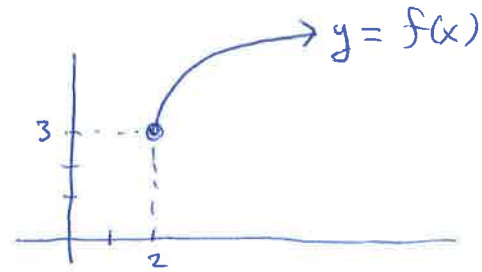
In class we concluded $\lim_{x \to 1} f(x)$ does not exist (DNE) because $f(x)$ approaches different values depending on whether x approaches c from the right or the left. Today we'll salvage some meaning from this situation by defining right- and left-hand limits.

$\lim_{x \to 1} f(x)$ DNE

$\lim_{x \to 1^+} f(x) = 4 \longleftarrow$ Right-hand limit: x specified to approach 1 from right.

$\lim_{x \to 1^-} f(x) = 2 \longleftarrow$ Left-hand limit: x specified to approach 1 from left.

Ex Consider $f(x) = \sqrt{x-2} + 3$



$\lim_{x \to 2} f(x)$ DNE. (because $f(x)$ is not even defined if x approaches 2 from left)

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \sqrt{x-2} + 3 = \sqrt{2-2} + 3 = \boxed{3}$

$\lim_{x \to 2^-} f(x)$ DNE.

Theorem $\lim_{x \to c} f(x) = L \iff \lim_{x \to c^+} f(x) = L$ and $\lim_{x \to c^-} f(x) = L$