Section 1.3 Review of Trig Functions (Continued)

Recall:
\[ \cos(\theta) = \text{x-coordinate of } P \]
\[ \sin(\theta) = \text{y-coordinate of } P \]

Notation
- OK to write \( \cos \theta \) for \( \cos(\theta) \).
  It's like writing \( f(x) \) for \( f(x) \)
- \( \sin^2 \theta \) means \( (\sin(\theta))^2 \), etc.

Picture gives following Identities  (True for any \( \theta \))
- \( \sin^2 \theta + \cos^2 \theta = 1 \)
- \( \sin(-\theta) = -\sin(\theta) \)
- \( \cos(-\theta) = \cos(\theta) \)
- \( \sin(\theta + 2\pi k) = \sin(\theta) \) \quad \text{for } k = 0, \pm 1, \pm 2, \ldots \)
- \( \cos(\theta + 2\pi k) = \cos(\theta) \)

More notation
Independent variable (of course) can be something besides \( \theta \).

\[ \cos(\theta), \cos(x), \cos(\pi), \cos(\frac{x^3 + 1}{1 + x^2}), \text{ etc.} \]
Six Trig Functions

\[ y = \sin(x) \]

\[ y = \cos(x) \]

\[ y = \tan(x) = \frac{\sin(x)}{\cos(x)} \]

\[ y = \cot(x) = \frac{\cos(x)}{\sin(x)} \]

\[ y = \sec(x) = \frac{1}{\cos(x)} \]

\[ y = \csc(x) = \frac{1}{\sin(x)} \]
Solving Trig Equations

Ex Find all solutions: \( \cos(x) = \frac{1}{2} \)

Solutions: \( x = \frac{\pi}{3} + 2\pi k \) \( k = 0, \pm 1, \pm 2, \ldots \)
\( x = -\frac{\pi}{3} + 2\pi k \)

Ex Find the domain: \( f(x) = \frac{1}{1 + \sin(x)} \)

Domain: \( \{ x \mid x \in \mathbb{R}, x \neq \frac{3\pi}{2} + 2k\pi \} \) (where \( k = 0, \pm 1, \pm 2, \ldots \))

Ex Solve: \( \cos^2 x - \cos x = 0 \)
\( \cos(x)(\cos(x) - 1) = 0 \)
\( \cos(x) = 0 \quad \cos(x) = 1 \)
\( x = \frac{\pi}{2} + k\pi \quad x = 2k\pi \)

Solutions: \( x = \frac{\pi}{2} + k\pi \) \( k = 0, \pm 1, \pm 2, \pm 3, \ldots \)
\( x = 2\pi k \)
More Identities

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]  
\[ \sin(\alpha + \beta) = \cos \alpha \sin \beta + \sin \alpha \cos \beta \]  
Addition Formulas

\[ \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha \]  
\[ \sin(2\alpha) = 2 \cos \alpha \sin \alpha \]  
Double Angle Formulas.

Solving Triangles

(finding lengths of missing sides, etc.)

By similar \( \triangle \)'s

\[ \frac{\sin \theta}{1} = \frac{\text{OPP}}{\text{HYP}} \]  
\[ \frac{\cos \theta}{1} = \frac{\text{ADJ}}{\text{HYP}} \]  
\[ \frac{\sin \theta}{\cos(\theta)} = \frac{\text{OPP}}{\text{ADJ}} \]  

\[ \tan \theta = \frac{\text{OPP}}{\text{ADJ}} \]

Example

Find the missing side \( a \).

\[ \frac{5}{a} = \frac{\text{OPP}}{\text{HYP}} = \tan \left( \frac{\pi}{2} \right) \]

\[ a = \frac{5}{\tan \left( \frac{\pi}{2} \right)} \approx 10.3826 \]