Chapter 45

(2) Note that the function $f(x) = e^{\cos(x)}\sqrt[3]{x}$ is odd because $f(-x) = e^{\cos(-x)}\sqrt[3]{-x} = e^{\cos(x)}(-\sqrt[3]{x})$

$= -e^{\cos(x)}\sqrt[3]{x} = -f(x)$. (Recall $\cos(-x) = \cos(x)$.)

Therefore $\int_{-1}^{1} e^{\cos(x)}\sqrt[3]{x}\,dx = \boxed{0}$

(14) Find the average value of $1/x$ on $[1,5]$

<u>Answer</u>: $\frac{1}{5-1}\int_1^5 \frac{1}{x}\,dx = \frac{1}{4}\left[\ln(x)\right]_1^5$

$= \frac{1}{4}\left(\ln(5) - \ln(1)\right) = \frac{1}{4}\ln(5) \approx 0.4023$

(24) Find the equation of the tangent line to $f(x) = \int_{-2}^{x} \frac{t^3}{\sqrt{t^2+5}}\,dt$ at the point $(2, f(2))$.

<u>Solution</u> $f(2) = \int_{-2}^{2} \frac{t^3}{\sqrt{t^2+5}}\,dt = 0$ because the integrand is <u>odd</u>. Thus the point is $(2,0)$

Also by FTC 1, $f'(x) = \frac{x^3}{\sqrt{x^2+5}}$, so the slope is $f'(2) = \frac{2^3}{\sqrt{2^2+5}} = \frac{8}{3}$. By the point-slope formula the equation is $y - 0 = \frac{8}{3}(x-2) = \frac{8}{3}x - \frac{16}{3}$ $\boxed{y = \frac{8}{3}x - \frac{16}{3}}$