Chapter 4.4

MATH 200

14 $\int \sin(x)\, e^{\cos(x)}\, dx$

Let $u = \cos(x)$

Then $\frac{du}{dx} = -\sin(x)$

So $-du = \sin(x)\,dx$

$= \int e^{\cos(x)} \sin(x)\,dx$

$= \int e^{u}(-du) = -\int e^{u}\,du = -e^{u} + C = \boxed{-e^{\cos(x)} + C}$

10 $\int \frac{1}{x^2}\sqrt{2 - \frac{1}{x}}\; dx$

Let $u = 2 - \frac{1}{x}$

Then $\frac{du}{dx} = \frac{1}{x^2}$

So $du = \frac{1}{x^2}\,dx$

$= \int \left(2 - \frac{1}{x}\right)^{1/2} \frac{1}{x^2}\,dx$

$= \int u^{1/2}\,du = \frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1} + C = \frac{1}{3/2} u^{3/2} + C = \frac{2u^{3/2}}{3} + C$

$= \frac{2\sqrt{u}^{\,3}}{3} + C = \boxed{\frac{2}{3}\sqrt{2 - \frac{1}{x}}^{\,3} + C}$

2 $\int e^{2x^2}\, 4x\, dx$

Let $u = 2x^2$

Then $\frac{du}{dx} = 4x$

So $du = 4x\,dx$

$= \int e^{u}\,du = e^{u} + C$

$= \boxed{e^{2x^2} + C}$

(26) $\displaystyle\int_1^2 \frac{x+1}{(x^2+2x)^2}\,dx$

Let $u = x^2+2x$

So $\dfrac{du}{dx} = 2x+2$

and $\frac{1}{2}\,du = (x+1)\,dx$

$$= \int_1^2 (x^2+2x)^{-2}(x+1)\,dx$$

$$= \int_{1^2+2\cdot 1}^{2^2+2\cdot 2} u^{-2}\,\frac{1}{2}\,du = \frac{1}{2}\int_3^8 u^{-2}\,du = \frac{1}{2}\left[\frac{1}{-2+1}u^{-2+1}\right]_3^8$$

$$= \frac{1}{2}\left[-\frac{1}{u}\right]_3^8 = \frac{1}{2}\left(-\frac{1}{8}-\left(-\frac{1}{3}\right)\right) = \frac{1}{2}\left(\frac{1}{3}-\frac{1}{8}\right) = \frac{1}{2}\left(\frac{8}{24}-\frac{3}{24}\right) = \boxed{\frac{5}{48}}$$

(30) $\displaystyle\int_0^1 x\sqrt{x^2+1}\,dx$

Let $u = x^2+1$

So $\dfrac{du}{dx} = 2x$

and $\frac{1}{2}\,du = x\,dx$

$$= \int_0^1 (x^2+1)^{\frac{1}{2}}\,x\,dx$$

$$= \int_{0^2+1}^{1^2+1} u^{\frac{1}{2}}\,\frac{1}{2}\,du = \frac{1}{2}\int_1^2 u^{\frac{1}{2}}\,du = \frac{1}{2}\left[\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{1}{3/2}u^{3/2}\right]_1^2 = \frac{1}{2}\left[\frac{2}{3}\sqrt{u}^3\right]_1^2 = \frac{1}{3}\left[\sqrt{u}^3\right]_1^2 = \frac{\sqrt{2}^3-\sqrt{1}^3}{3}$$

$$= \boxed{\frac{2\sqrt{2}-1}{3}}$$