Chapter 43

MATH 200

(2) Method I By FTC 1, $\frac{d}{dx}\left[\int_2^x (t^5+e^t+3)\,dt\right] = \boxed{x^5+e^x+3}$

Method II $\frac{d}{dx}\left[\int_2^x (t^5+e^t+3)\,dt\right] = \frac{d}{dx}\left[\left[\frac{t^6}{6}+e^t+3t\right]_2^x\right]$

$= \frac{d}{dx}\left[\left(\frac{x^6}{6}+e^x+3x\right)-\left(\frac{2^6}{6}+e^2+3\cdot 2\right)\right]$

$= \frac{d}{dx}\left[\frac{x^6}{6}+e^x+3x-\frac{64}{6}-e^2-6\right] = \boxed{x^5+e^x+3}$

(6) $\frac{d}{dx}\left[\int_0^x \sqrt{10+\sin(t)+\cos(t)}\,dt\right] = \boxed{\sqrt{10+\sin(x)+\cos(x)}}$

$\frac{d}{dx}\left[\int_x^0 \sqrt{10+\sin(t)+\cos(t)}\,dt\right] = \frac{d}{dx}\left[-\int_0^x \sqrt{10+\sin(t)+\cos(t)}\,dt\right]$

$= \boxed{-\sqrt{10+\sin(x)+\cos(x)}}$

(8) $y = \int_1^{x^2-x+2} \cos\left(\frac{t}{1+t}\right)dt.$ This is a composition:

$\begin{cases} y = \int_1^u \cos\left(\frac{t}{1+t}\right)dt \\ u = x^2-x+2 \end{cases}$ Then by chain rule:

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \cos\left(\frac{u}{1+u}\right)(2x-1)$

$= \cos\left(\frac{x^2-x+2}{1+x^2-x+2}\right)(2x-1)$

Chapter 43

(14) Differentiate $\int_{-x}^{x} \sin(t^2-2)t^2\,dt$

Solution $\int_{-x}^{x} \sin(t^2-2)t^2\,dt = \int_{-x}^{0} \sin(t^2-2)t^2\,dt + \int_{0}^{x} \sin(t^2-2)t^2\,dt$

$= -\int_{0}^{-x} \sin(t^2-2)t^2\,dt + \int_{0}^{x} \sin(t^2-2)t^2\,dt.$

Applying FTC 1 and the chain rule, the derivative is $-\sin((-x)^2-2)(-x)^2(-1) + \sin(x^2-2)x^2$

$= \sin(x^2-2)x^2 + \sin(x^2-2)x^2 = \boxed{2\sin(x^2-2)x^2}$

(16) Find the equation of the tangent line to $f(x) = \int_{-2}^{x} \frac{t^3}{\sqrt{t^2+5}}\,dt$ at the point $(-2, f(-2))$

Solution Notice that $f(-2) = \int_{-2}^{-2} \frac{t^3}{\sqrt{t^2+5}}\,dt = 0,$

so we are seeking the tangent at the point $(-2,0)$.

Now $f'(x) = \frac{x^3}{\sqrt{x^2+5}}$ by FTC 1, and then

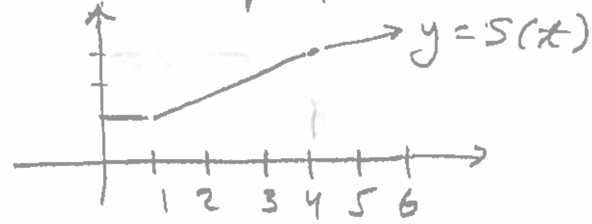
$f'(-2) = \frac{(-2)^3}{\sqrt{(-2)^2+5}} = \frac{-8}{\sqrt{9}} = -\frac{8}{3}$. So the tangent line has slope $-8/3$ and passes through $(-2,0)$.

By the point-slope formula its equation is

$y - y_0 = m(x-x_0) \rightarrow y - 0 = \frac{-8}{3}(x-(-2)) \rightarrow \boxed{y = \frac{-8}{3}x - \frac{16}{3}}$

Chapter 43

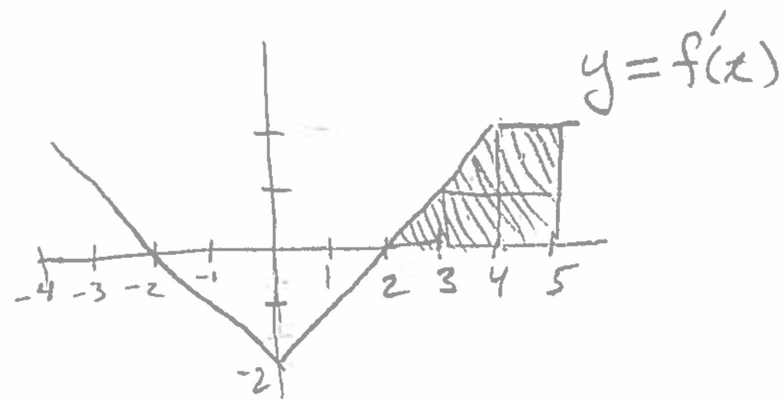
(22) An object has velocity $v(t)$ at time t.
Its position function $s(t)$ is graphed below.
Find $\int_0^4 v(t)\,dt$



Answer Because $s(t)$ is an antiderivative of $v(t)$,

$$\int_0^4 v(t)\,dt = s(4) - s(0) = 3 - 1 = \boxed{2}$$

(24) The derivative of a function $f(x)$ is graphed.
If $f(5) = 3$, what is $f(2)$?



Answer $$\int_2^5 f'(t)\,dt = f(5) - f(2)$$

By area, $\int_2^5 f'(t) = 4$. Thus the above gives $4 = f(5) - f(2)$, $4 = 3 - f(2)$, so $\boxed{f(2) = -1}$