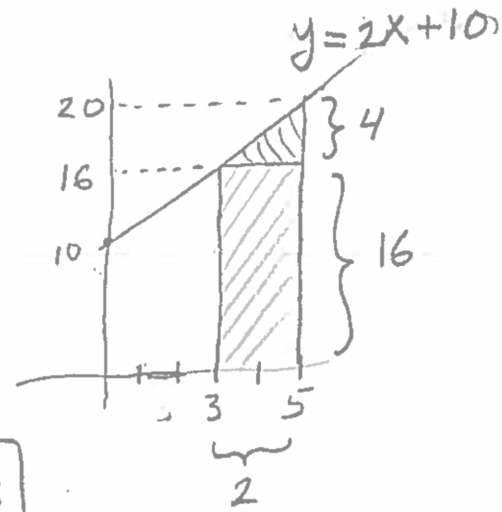
(4) Here $\Delta x = \frac{5-1}{n} = \frac{4}{n}$ and $x_k = 1 + k\Delta x = 1 + \frac{4k}{n}$

$$\int_1^5 (x^2+x+10)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$
$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(1+\frac{4k}{n}\right)\frac{4}{n}$$
$$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(1+\frac{4k}{n}\right)^2 + \left(1+\frac{4k}{n}\right) + 10\right)\frac{4}{n}$$

(6) $\int_3^5 (2x+10)\,dx$



$= (\text{area of rectangle}) + (\text{area of } \Delta)$

$= 2\cdot 16 + \frac{1}{2}\, 2\cdot 4$

$= 32 + 4 = \boxed{36 \text{ square units}}$

(16) Suppose $\int_0^5 f(x)\,dx = 3$, $\int_0^2 3g(x)\,dx = 12$, $\int_2^5 g(x)\,dx = -1$,

Find $\int_0^5 (3f(x) + g(x))\,dx$.

From $\int_0^2 3g(x)\,dx = 12$, get $3\int_0^2 g(x)\,dx = 12$, so $\boxed{\int_0^2 g(x)\,dx = 4}$

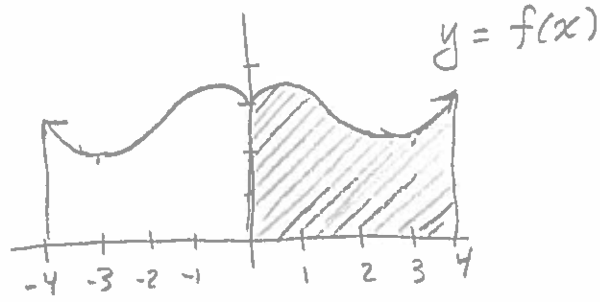
Also $\int_0^5 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^5 g(x)\,dx = 4 - 1 = \boxed{3}$

Then $\int_0^5 (3f(x)\,dx + g(x))\,dx = 3\int_0^5 f(x)\,dx + \int_0^5 g(x)\,dx$

$= 3\cdot 3 + 3 = \boxed{12}$

Chapter 41

(20) The graph of $f(x)$ is symmetric with respect to the y-axis. Thus



$$\int_0^4 f(x)\,dx = \frac{1}{2}\int_{-4}^{4} f(x)\,dx = \frac{1}{2}\,22.6 = \boxed{11.3}$$

(22) Write $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{2+\frac{7k}{n}}{1+\left(2+\frac{7k}{n}\right)^2}\,\frac{7}{n}$ as a definite integral.

As k goes from 0 to n, $0, 1, 2, 3, \cdots, n$

the expression $2+\frac{7k}{n}$ takes on the values

$$2 = 2+\frac{7\cdot 0}{n},\quad 2+\frac{7}{n},\quad 2+\frac{7\cdot 2}{n},\quad 2+\frac{7\cdot 3}{n},\quad \cdots,\quad 2+\frac{7\cdot n}{n} = 9$$

These are evenly spaced from $a=2$ to $b=9$, and $\Delta x = \frac{b-a}{n} = \frac{9-2}{n} = \frac{7}{n}$. Let $x_k = 2+\frac{7\cdot k}{n}$.

Then the limit is

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x_k}{1+x_k^2}\,\Delta x = \boxed{\int_2^9 \frac{x}{1+x^2}\,dx}$$