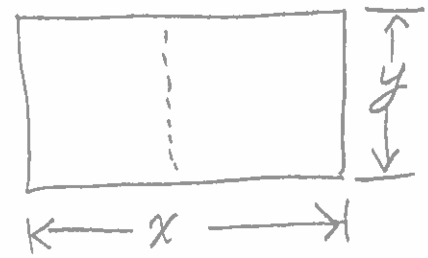
(2) A total of 2000 square feet is to be enclosed by two pens.
Outside walls: brick (\$10 per ft)
Inside wall: chain link (\$5 per ft.)
Find the dimensions x and y that minimize cost of materials.



Solution Constraint is area $= xy = 2000$, so $\boxed{y = \frac{2000}{x}}$.

We wish to minimize cost, which is as follows.

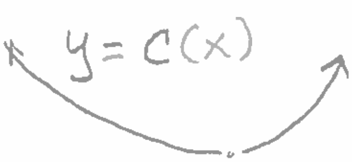
$$\text{Cost} = 10(\#\text{ feet of brick wall}) + 5(\#\text{ feet of chain link})$$
$$= 10(2x+2y) + 5y = 20x + 25y$$
$$= 20x + 25\frac{2000}{x} = 20x + \frac{50000}{x}.$$

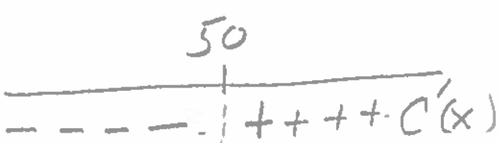
Thus we seek the x that minimizes

$C(x) = 20x + \frac{50000}{x}$ on the interval $(0, \infty)$.

$$C'(x) = 20 - \frac{50000}{x^2} = 0$$
$$20 = \frac{50000}{x^2}$$
$$20x^2 = 50000$$
$$x^2 = 2500$$
$$x = 50$$

critical point



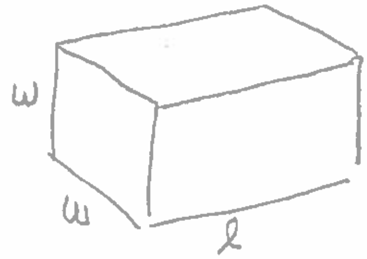


Global minimum cost is at $x = 50$
Then $y = \frac{2000}{50} = 40$

Answer: Use dimensions $x = 50$, $y = 40$

Chapter 34

⑥ Length plus girth of a USPS package can't exceed 108".
You need to mail a box whose width (w) and height are equal, and with greatest possible volume. Find the dimensions $w \times w \times l$ of the package.

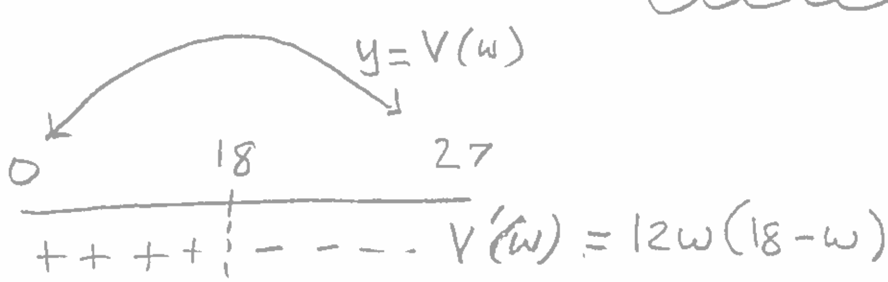


Solution Constraint is length + girth = 108, meaning
$l + 4w = 108$, so $\boxed{l = 108 - 4w}$

We want to maximize Volume $= wwl = w^2(108-4w)$
$= 108w^2 - 4w^3$.

Thus we seek maximum of $V(w) = 108w^2 - 4w^3$ on the interval $(0, \frac{108}{4}) = (0, 27)$ ← $\frac{108}{4}$ because girth $4w$ can't exceed 108

$V'(w) = 216w - 12w^2$
$= 12w(18-w)$ ← Critical points are 0 and 18 but 0 is not in $(0, 27)$



We have a global maximum for volume at $w = 18$, so $l = 108 - 4 \cdot 18 = 36$

$\boxed{\text{Answer To maximize volume use dimensions } 18 \times 18 \times 36}$