(2) Find global extrema of $f(x) = x^3 - 3x$ on $[0, 2]$

$f'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1) = 0$

$\boxed{\text{Critical pts: } x = 1 \quad x = -1}$

The only critical point in $[0, 2]$ is $x = 1$

$f(0) = 0^3 - 3\cdot 0 = 0$
$f(1) = 1^3 - 3\cdot 1 = -2$
$f(2) = 2^3 - 3\cdot 2 = 2$

Global minimum is $f(1) = -2$ at $x = 1$
Global maximum is $f(2) = 2$ at $x = 2$.

(4) Find global extrema of $f(x) = \sqrt[3]{x}\,(x-8)$ on $[-1, 27]$

$f(x) = x^{1/3}(x-8) = x^{4/3} - 8x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} - \frac{8}{3}x^{-2/3} = \frac{4}{3}\sqrt[3]{x} - \frac{8}{3\sqrt[3]{x}^2} = \frac{4}{3}\left(\sqrt[3]{x} - \frac{2}{\sqrt[3]{x}^2}\right)$

$= \frac{4}{3}\,\frac{x-2}{\sqrt[3]{x^2}}$ ← critical points are 0, 2

$f(-1) = \sqrt[3]{-1}(-1-8) = 9$
$f(0) = \sqrt[3]{0}(0-8) = 0$
$f(2) = \sqrt[3]{2}(2-8) = -6\sqrt[3]{2}$ ← global min.
$f(27) = \sqrt[3]{27}(27-8) = 3\cdot 19 = 57$ ← global max.

Global minimum is $f(2) = -6\sqrt[3]{2}$ at $x = 2$
Global maximum is $f(27) = 57$ at $x = 27$

(8) Find the global extrema of $f(x) = 2\sqrt{x} - x$ on the <u>open</u> interval $(0, 4)$.

$$f(x) = 2x^{\frac{1}{2}} - x$$

$$f'(x) = x^{-\frac{1}{2}} - 1 = \frac{1}{\sqrt{x}} - 1$$

$f'(x)$ is not defined at $x=0$, but this critical point is <u>not</u> in $(0, 4)$.
To find other critical points, solve

$$f'(x) = 0$$

$$\frac{1}{\sqrt{x}} - 1 = 0$$

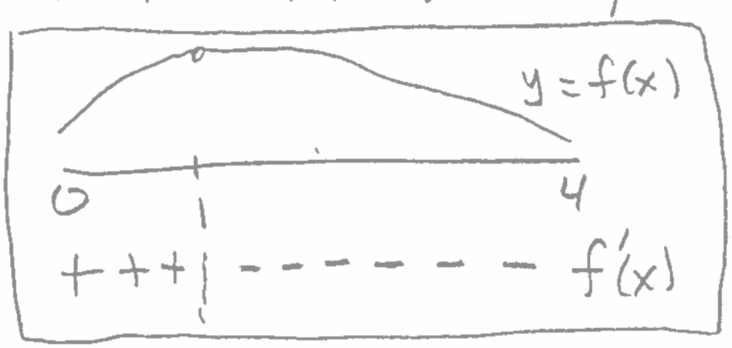
$$\frac{1}{\sqrt{x}} = 1$$

$$1 = \sqrt{x}$$

$$x = 1$$

critical point.

Thus there is <u>only one</u> critical point $x=1$ in the interval $(0, 4)$. By the 1st derivative test, there is a local max at $x=1$



Thus $f(x)$ has a global max at $x=1$.
No global minimum