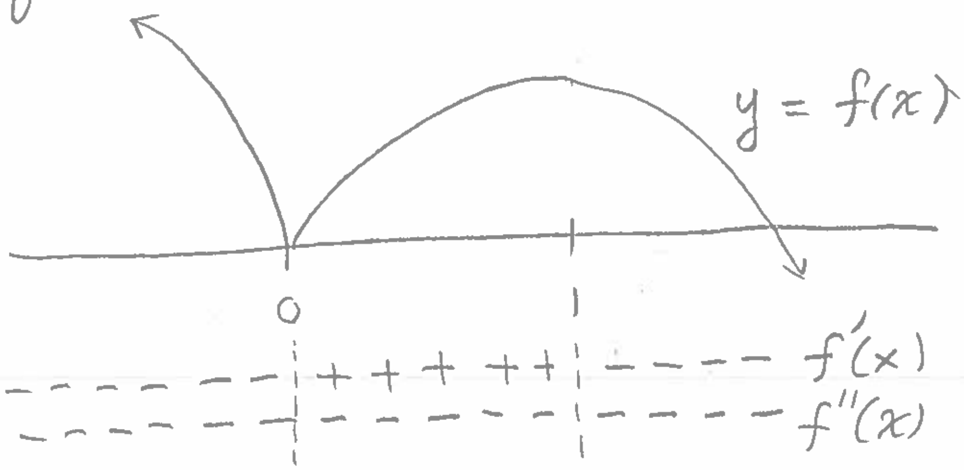
Chapter 32 MATH 200

② Find intervals of increase/decrease and concavity:

$$f(x) = 3\sqrt[3]{x}^{\,2} - 2x = 3x^{2/3} - 2x$$

$$f'(x) = 3\cdot\frac{2}{3}x^{-1/3} - 2 = \frac{2}{\sqrt[3]{x}} - 2 = 2\left(\frac{1}{\sqrt[3]{x}} - 1\right)$$

Critical points are $x=0$ (because $f'(0)$ not defined) and $x=1$ (because $f'(1)=0$.)



Test points:

$f'(-1) = -4 < 0$

$f'(\tfrac{1}{8}) = 2 > 0$

$f'(8)\ -1 < 0$

$$f''(x) = -\frac{2}{3}x^{-1/3-1} = -\frac{2}{3}x^{-4/3} = -\frac{2}{3\sqrt[3]{x}^{\,4}}$$

Always negative but undefined at $x=0$

Answers:

$f(x)$ increases on $(0,1)$

$f(x)$ decreases on $(-\infty, 0)$ and $(1,\infty)$

$f(x)$ concave down on $(-\infty,0)\cup(0,\infty)$

(and concave up nowhere)

Chapter 32

(6) Use 2^{nd} derivative test to find local extrema of $f(x) = x^2 e^{-x}$

$$f'(x) = 2xe^{-x} + x^2 e^{-x}(-1) = xe^{-x}(2-x)$$

This is defined for all x and equals 0 if $x=0$ or $x=2$. Thus critical points are 0 and 2.

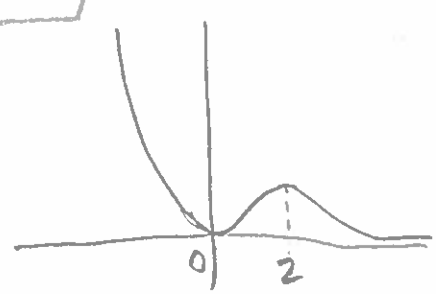
To find f'' write $f'(x) = (2x - x^2)e^{-x}$.

$$f''(x) = (2-2x)e^{-x} + (2x - x^2)e^{-x}(-1)$$
$$= e^{-x}(2 - 4x + x^2)$$

$f''(0) = e^0(2 - 4\cdot 0 + 0^2) = 2 > 0$ (local min at 0)

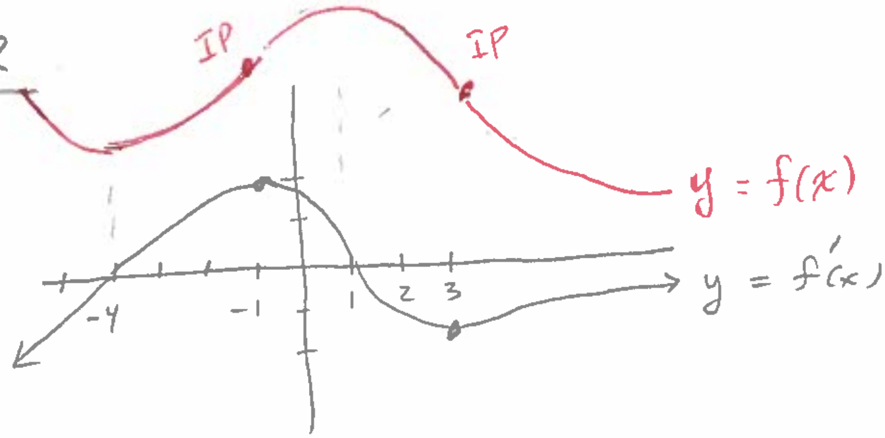
$f''(2) = e^2(2 - 4\cdot 2 + 2^2) = -2e^2 < 0$ (local max at 2)

By 2^{nd} derivative test $\boxed{f \text{ has a local min at } x=0 \text{ and a local max at } x=2.}$



Chapter 32

(14)



(a) f increases on $(-4, 1)$ because $f'(x) > 0$ there.
f decreases on $(-\infty, -4) \cup (1, \infty)$ since $f'(x) < 0$ there.

(b) Critical points of f are -4 and 1 because $f'(-4) = 0$ and $f'(1) = 0$.

(c) f has local min at $x = -4$ because $f'(x)$ changes from $-$ to $+$ there.
f has local max at $x = 3$ because $f'(x)$ changes from $+$ to $-$ there.

(d) f concave up on $(-\infty, -1)$ and $(3, \infty)$ because f' increase there, so $f''(x) > 0$.

f concave down on $(-1, 3)$ because f' decreases there and so $f''(x) < 0$.

(e) Possible graph of f(x) is sketched above.