(2) A spherical balloon is inflated at a rate of 100π cubic feet per min. In Exercise 1 you found that when the radius r is 5 feet, the radius is increasing at a rate of 1 ft/min. Now find how fast surface area S is increasing at the instant $r = 5$.

Solution We know $\frac{dr}{dt} = 1$ (when $r=5$)

We want $\frac{dS}{dt}$ (when $r = 5$)

Formula for surface area: $S = 4\pi r^2$

$$\frac{d}{dt}[S] = \frac{d}{dt}\left[4\pi r^2\right]$$

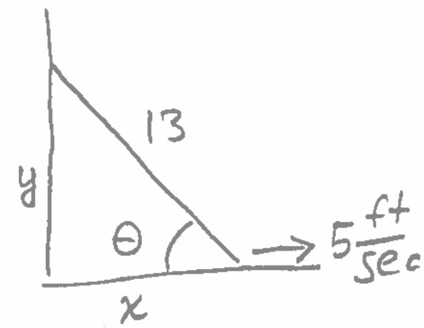
$$\frac{dS}{dt} = 4\pi \, 2r^1 \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi \cdot 5 \cdot 1$$

$$= \boxed{40\pi \text{ sq ft/min}}$$

(4) A 13-foot ladder is leaning against a wall as shown. Its base slides away from the wall at a rate of 5 ft/sec. At what rate is angle θ changing when the base is 12 feet from the wall?



Know $\frac{dx}{dt} = 5$

Want $\frac{d\theta}{dt}$

$$\cos(\theta) = \frac{\text{ADJ}}{\text{HYP}} = \frac{x}{13}$$

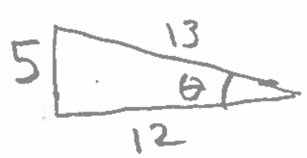
$$\frac{d}{dt}\left[\cos(\theta)\right] = \frac{d}{dt}\left[\frac{x}{13}\right]$$

$\frac{dx}{dt} = 5$

$$-\sin(\theta)\frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{13}\frac{dx}{dt}\frac{-1}{\sin(\theta)} = -\frac{5}{13}\csc(\theta)$$

$$= -\frac{5}{13}\frac{\text{HYP}}{\text{OPP}} = -\frac{5}{13}\frac{13}{5} = \boxed{-1 \text{ radians/sec}}$$



When $x = 12$, $y = 5$