Chapter 19

MATH 200

(6) $f(x) = \frac{3}{x^4} + \frac{e^x}{3} = 3x^{-4} + \frac{1}{3}e^x$

$f'(x) = -12x^{-4-1} + \frac{1}{3}e^x = \boxed{-\frac{12}{x^5} + \frac{e^x}{3}}$

(8) Find all x for which the tangent to $y = e^x - x$ is horizontal at $(x, e^x - x)$.

Note that $\frac{dy}{dx} = e^x - 1$, so we seek all x for which $e^x - 1 = 0$. Solving this:

$$e^x = 1$$

$$\ln(e^x) = \ln(1)$$

$$x = 0$$

Conclusion The tangent to $y = e^x - x$ is horizontal only at $x = 0$.

Chapter 20

(2) $\frac{d}{dx}\left[e^x\sqrt{x}\right] = \frac{d}{dx}\left[e^x x^{\frac{1}{2}}\right] = e^x x^{\frac{1}{2}} + e^x \frac{1}{2} x^{-\frac{1}{2}} = \boxed{e^x\sqrt{x} + \frac{e^x}{2\sqrt{x}}}$

(8) $\frac{d}{dx}\left[\frac{x^3+x^2+1}{x}\right] = \frac{(3x^2+2x)x - (x^3+x^2+1)\cdot 1}{x^2}$

$= \boxed{\frac{2x^3+x^2-1}{x^2}}$

Chapter 20 Continued

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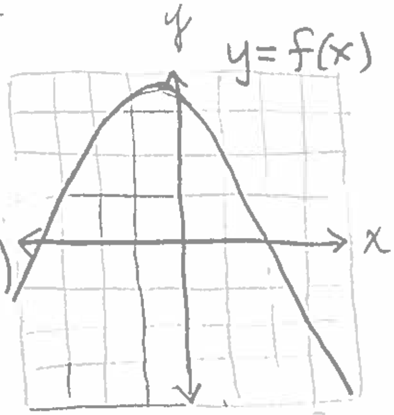

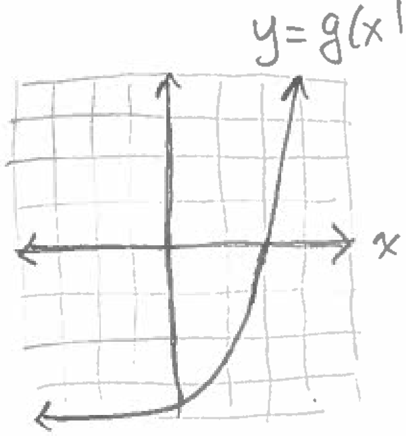

(a) $h(x) = f(x)g(x)$

$h'(x) = f'(x)g(x) + f(x)g'(x)$

Therefore

$h'(1.5) = f'(1.5)g(1.5) + f(1.5)g'(1.5)$

$= (-2)(-2) + 1 \cdot 4 = \boxed{8}$

(b) $h(x) = \dfrac{f(x)}{g(x)}$ so $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

and $h'(1.5) = \dfrac{f'(1.5)g(1.5) - f(1.5)g'(1.5)}{(g(1.5))^2} = \dfrac{(-2)(-2) - 1 \cdot 4}{(-2)^2} = \boxed{0}$

(c) $h(x) = \dfrac{g(x)}{f(x)}$ so $h'(x) = \dfrac{g'(x)f(x) - g(x)f'(x)}{(f(x))^2}$

and $h'(1.5) = \dfrac{g'(1.5)f(1.5) - g(1.5)f'(1.5)}{(f(1.5))^2} = \dfrac{4 \cdot 1 - (-2)(-2)}{1^2} = \boxed{0}$

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(a) $h(x) = (x + x^3)f(x)$

$h'(x) = (1 + 2x^2)f(x) + (x + x^3)f'(x)$

$h'(3) = (1 + 2 \cdot 3^2) \cdot f(3) + (3 + 3^3)f'(3)$

$= 19 \cdot 1 + 30 \cdot 3 = \boxed{109}$

(b) $h(x) = \dfrac{g(x)}{f(x)}$ $\quad h'(3) = \dfrac{g'(3)f(3) - g(3)f'(3)}{(f(3))^2} = \dfrac{-3 \cdot 1 - 4 \cdot 3}{1^2} = \boxed{-15}$

(c) $h(x) = f(x)g(x)$ $\quad h'(3) = g'(3)f(3) + g(3)f'(3) = -3 \cdot 1 + 4 \cdot 3$

$= \boxed{9}$

(2) $f(r) = 3e^r - \cos(r) + \frac{1}{r}$

$$= 3e^r - \cos(r) + r^{-1}$$

$$f'(r) = 3e^r + \sin(r) - r^{-1-1} = \boxed{3e^r + \sin(r) - \frac{1}{r^2}}$$

(4) $y = x^4 \tan(x)$

$$\boxed{\frac{dy}{dx} = 4x^3 \tan(x) + x^4 \sec^2(x)}$$ (product rule)

(14) $y = \dfrac{x\cos(x)}{\sin(x)+1}$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}\left[x\cos(x)\right](\sin(x)+1) - x\cos(x)\frac{d}{dx}\left[\sin(x)+1\right]}{(\sin(x)+1)^2}$$

$$= \boxed{\frac{(\cos(x) - x\sin(x))(\sin(x)+1) - x\cos(x)\cos(x)}{(\sin(x)+1)^2}}$$

(18) Let $f(x) = \tan(x)$. Then

$$\lim_{h\to 0} \frac{\tan(x+h) - \tan(x)}{h} = f'(x) = \sec^2(x)$$

Therefore $\displaystyle\lim_{h\to 0} \frac{\tan(\frac{\pi}{4}+h) - \tan(\frac{\pi}{4})}{h} = f'\left(\frac{\pi}{4}\right)$

$$= \sec^2\left(\frac{\pi}{4}\right) = \left(\frac{1}{\cos(\frac{\pi}{4})}\right)^2 = \left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2 = \boxed{2}$$