② Find all x for which tangent to $f(x) = 2\sqrt{x} - x$ is horizontal

Note: $f(x) = 2x^{\frac{1}{2}} - x$. We need to solve $f'(x) = 0$.

$$2\,\tfrac{1}{2}x^{\frac{1}{2}-1} - 1 = 0$$
$$x^{-\frac{1}{2}} - 1 = 0$$
$$\frac{1}{\sqrt{x}} = 1$$
$$1 = \sqrt{x}$$
$$1^2 = \sqrt{x}^{\,2}$$
$$1 = x$$

Answer: Tangent to $y = f(x)$ at $(1, f(1)) = (1,1)$ is horizontal

④ Find all x for which the tangent to $f(x) = x^3 - 3x$ at $(x, f(x))$ is parallel to the tangent to $g(x) = 3x^2 + 6x$ at $(x, g(x))$

Need to solve $f'(x) = g'(x)$

$$3x^2 - 3 = 6x + 6$$
$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$3(x+1)(x-3) = 0$$

Answer: Tangents are parallel at $x = -1$ and $x = 3$

⑥ Find the equation of the tangent to $y = \sqrt{x} = x^{\frac{1}{2}}$ at $(9,3)$.

Note: $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$. Thus tangent at $(9,3)$ has slope $f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{6}$. By point-slope formula the equation of the tangent is

$$y - y_0 = m(x - x_0)$$
$$y - 3 = \frac{1}{6}(x - 9)$$
$$y = \frac{1}{6}x - \frac{3}{2} + 3$$
$$\boxed{y = \frac{1}{6}x + \frac{3}{2}}$$

⑫