Ch 15

MATH 200

④ Find the slope of the tangent to $f(x)=\frac{1}{x^2}$ at $(-2, \frac{1}{4})$.

$$m = \lim_{z\to a} \frac{f(z)-f(a)}{z-a}$$

$$= \lim_{z\to -2} \frac{\frac{1}{z^2}-\frac{1}{(-2)^2}}{z-(-2)} = \lim_{z\to 2} \frac{\frac{1}{z^2}-\frac{1}{4}}{z+2}$$

$$= \lim_{z\to -2} \frac{\frac{1}{z^2}-\frac{1}{4}}{z+2}\cdot\frac{4z^2}{4z^2} = \lim_{z\to -2} \frac{4-z^2}{(z+2)4z^2}$$

$$= \lim_{z\to -2} \frac{(2-z)(2+z)}{(z+2)4z^2} = \lim_{z\to -2} \frac{2-z}{4z^2} = \frac{2-(-2)}{4(-2)^2} = \boxed{\frac{1}{4}}$$

Ch 16

④ Find the derivative of $f(x)=\sqrt{6x}$

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{\sqrt{6z}-\sqrt{6x}}{z-x}$$

$$= \lim_{z\to x} \frac{\sqrt{6z}-\sqrt{6x}}{z-x}\cdot\frac{\sqrt{6z}+\sqrt{6x}}{\sqrt{6z}+\sqrt{6x}}$$

$$= \lim_{z\to x} \frac{\sqrt{6z}^2+\sqrt{6z}\sqrt{6x}-\sqrt{6x}\sqrt{6z}-\sqrt{6x}^2}{(z-x)(\sqrt{6z}+\sqrt{6x})}$$

$$= \lim_{z\to x} \frac{6z-6x}{(z-x)(\sqrt{6z}+\sqrt{6x})} = \lim_{z\to x} \frac{6(z-x)}{(z-x)(\sqrt{6z}+\sqrt{6x})}$$

$$= \lim_{z\to x} \frac{6}{\sqrt{6z}+\sqrt{6x}} = \frac{6}{\sqrt{6x}+\sqrt{6x}} = \frac{6}{2\sqrt{6x}} = \frac{3}{\sqrt{6x}}$$

Thus $\boxed{f'(x)=\frac{3}{\sqrt{6x}}}$