Chapter 13

MATH 200

(2) $\lim_{x\to\infty} e^{\frac{1}{x}} = e^{\lim_{x\to\infty}\frac{1}{x}} = e^0 = \boxed{1}$

(6) Note: $\dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \dfrac{\sqrt{x}^2-\sqrt{2}^2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \dfrac{\cancel{(\sqrt{x}-\sqrt{2})}(\sqrt{x}+\sqrt{2})}{\sqrt{x}\,\cancel{(\sqrt{x}-\sqrt{2})}} = \dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}}$

(We will use this below.)

If $x \neq 2$.

(a) $\lim_{x\to 8} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \dfrac{8-2}{\sqrt{8}(\sqrt{8}-\sqrt{2})} = \dfrac{6}{2\sqrt{2}(2\sqrt{2}-\sqrt{2})} = \dfrac{6}{4} = \boxed{\dfrac{3}{2}}$

(b) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim_{x\to 2}\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}} = \dfrac{\sqrt{2}+\sqrt{2}}{\sqrt{2}} = \boxed{2}$

(c) $\lim_{x\to 0^+} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim_{x\to 0^+}\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}} = \boxed{\infty}$

approaches $\sqrt{2}$

approaches 0, positive

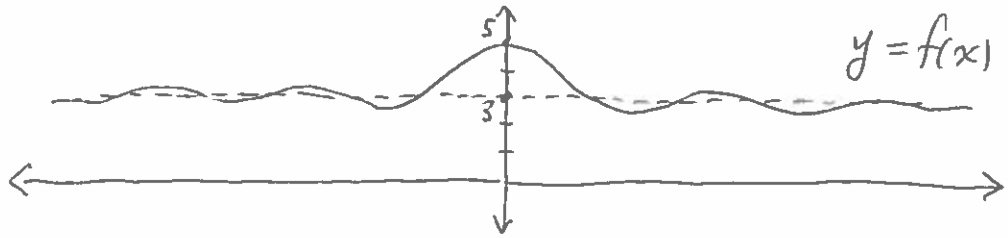
(d) $\lim_{x\to\infty} \dfrac{x-2}{\sqrt{x}(\sqrt{x}-\sqrt{2})} = \lim_{x\to\infty}\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}}$

$= \lim_{x\to\infty}\left(1+\dfrac{\sqrt{2}}{\sqrt{x}}\right) = (1+0) = \boxed{1}$

$\dfrac{\sqrt{2}}{\sqrt{x}}$ approaches 0 as $x\to\infty$

Chapter 13 (Continued)

(8)



(a) $\lim\limits_{x \to \infty} f(x) = \boxed{3}$

(b) $\lim\limits_{x \to -\infty} f(x) = \boxed{3}$

(c) $\lim\limits_{x \to \infty} \ln(4 - f(x)) = \ln\left(\lim\limits_{x \to \infty}(4 - f(x))\right)$

$= \ln(4-3) = \ln(1) = \boxed{0}$

(d) $\lim\limits_{x \to \infty} f\left(\frac{1}{x}\right) = f\left(\lim\limits_{x \to \infty} \frac{1}{x}\right) = f(0) = \boxed{5}$

(e) $\lim\limits_{x \to \infty} \dfrac{1}{f(x)} = \dfrac{\lim\limits_{x \to \infty} 1}{\lim\limits_{x \to \infty} f(x)} = \boxed{\dfrac{1}{3}}$

(f) $\lim\limits_{x \to \infty} \dfrac{1}{f\left(\frac{1}{x}\right)} = \dfrac{\lim\limits_{x \to \infty} 1}{\lim\limits_{x \to \infty} f\left(\frac{1}{x}\right)}$

$= \dfrac{1}{f\left(\lim\limits_{x \to \infty} \frac{1}{x}\right)} = \dfrac{1}{f(0)} = \boxed{\dfrac{1}{5}}$

Chapter 13 (Continued)

(18) Find the vertical and horizontal asymptotes of $f(x) = \dfrac{15 - 12x - 3x^2}{50 - 2x^2}$

In # 18 in Chapter 12 we saw that there is only one vertical asymptote, $\boxed{\text{the line } x = 5}$

For horizontal asymptotes, note that:

$$\lim_{x \to \infty} \frac{15 - 12x - 3x^2}{50 - 2x^2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\lim_{x \to \infty} \frac{15 - 12x - 3x^2}{50 - 2x^2} = \frac{-3}{-2} = \frac{3}{2}$$

Therefore the line $\boxed{y = \frac{3}{2} \text{ is a horizontal asymptote}}$