Section 5.4

8) $$\int_{-200}^{200} 2x^5\,dx = \boxed{0}$$ (because the integrand is odd)

22) Find the average value of $f(x) = x^2+1$ on $[-2, 2]$.

Ans: $$\frac{1}{2-(-2)}\int_{-2}^{2}(x^2+1)\,dx = \frac{1}{4}\left[\frac{x^3}{3}+x\right]_{-2}^{2}$$

$$= \frac{1}{4}\left(\frac{2^3}{3}+2\right) - \frac{1}{4}\left(\frac{(-2)^3}{3}+(-2)\right) = \frac{1}{4}\left(\frac{8}{3}+2\right) - \frac{1}{4}\left(-\frac{8}{3}-2\right)$$

$$= \frac{2}{3}+\frac{1}{2}+\frac{2}{3}+\frac{1}{2} = \frac{4}{3}+1 = \frac{4}{3}+\frac{3}{3} = \boxed{\frac{7}{3}}$$

Section 5.5

14) $$\int 8x\cos(4x^2+3)\,dx = \int \cos(4x^2+3)\,8x\,dx$$

$u = 4x^2+3$

$\frac{du}{dx} = 8x$

$du = 8x\,dx$

$$= \int \cos(u)\,du = \sin(u)+C$$

$$= \boxed{\sin(4x^2+3)+C}$$

18) $$\int x e^{x^2}\,dx = \int e^{x^2} x\,dx = \frac{1}{2}\int e^{x^2} 2x\,dx = \frac{1}{2}\int e^{u}\,du$$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$$= \frac{1}{2}e^{u}+C = \boxed{\frac{1}{2}e^{x^2}+C}$$

20) $$\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx = \int (\sqrt{x}+1)^4\frac{1}{2\sqrt{x}}\,dx = \int u^4\,du$$

$u = \sqrt{x}+1$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$$= \frac{u^5}{5}+C = \boxed{\frac{(\sqrt{x}+1)^5}{5}+C}$$

(22) $\displaystyle\int \frac{1}{10x-3}\,dx = \frac{1}{10}\int \frac{1}{10x-3}\,10\,dx = \frac{1}{10}\int \frac{1}{u}\,du$

$u = 10x-3$

$\frac{du}{dx} = 10$

$du = 10\,dx$

$$= \frac{1}{10}\ln|u| + C$$

$$= \boxed{\frac{1}{10}\ln|10x-3| + C}$$

(30) $\displaystyle\int \frac{3}{1+25y^2}\,dy = 3\int \frac{1}{1+(5y)^2}\,dy = \frac{3}{5}\int \frac{1}{1+(5y)^2}\,5\,dy$

$u = 5y$

$\frac{du}{dy} = 5$

$du = 5\,dy$

$$= \frac{3}{5}\int \frac{1}{1+u^2}\,du = \frac{3}{5}\tan^{-1}(u) + C$$

$$= \boxed{\frac{3}{5}\tan^{-1}(5y) + C}$$

(40) $\displaystyle\int_0^2 \frac{2x}{(x^2+1)^2}\,dx = \int_0^2 (x^2+1)^{-2}\,2x\,dx = \int_{0^2+1}^{2^2+1} u^{-2}\,du$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$$= -u^{-1}\Big|_1^5 = -\frac{1}{5} - \left(-\frac{1}{1}\right) = -\frac{1}{5} + 1 = \boxed{\frac{4}{5}}$$

(46) $\displaystyle\int_0^{\pi/4} \frac{\sin(\theta)}{\cos^3(\theta)}\,d\theta = -\int_0^{\pi/4} (\cos(\theta))^{-3}(-\sin(\theta))\,d\theta$

$u = \cos(\theta)$

$\frac{du}{d\theta} = -\sin(\theta)$

$du = -\sin(\theta)\,d\theta$

$$= -\int_{\cos(0)}^{\cos(\pi/4)} u^{-3}\,du = \frac{1}{2u^2}\Big|_1^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{2\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{1}{2\cdot 1^2} = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$$

(52) $\displaystyle\int_0^{\ln 4} \frac{e^x}{3+2e^x}\,dx = \frac{1}{2}\int_0^{\ln 4} \frac{1}{3+2e^x}\,2e^x\,dx$

$u = 3 + 2e^x$

$\frac{du}{dx} = 2e^x$

$du = 2e^x\,dx$

$$= \frac{1}{2}\int_{3+2e^0}^{3+2e^{\ln 4}} \frac{1}{u}\,du = \frac{1}{2}\int_5^{11} \frac{1}{u}\,du$$

$$= \frac{1}{2}\Big[\ln|u|\Big]_5^{11} = \frac{1}{2}(\ln 11 - \ln 5) = \boxed{\frac{1}{2}\ln\frac{11}{5}}$$