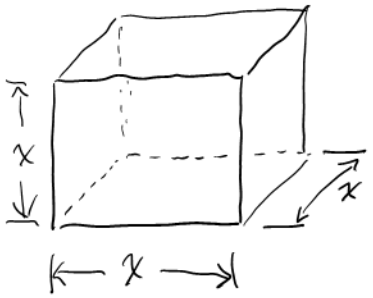
Section 3.11

(10) Edges of a cube increase at 2 cm/sec. How fast is volume changing when each edge is 50 cm?



Know $\frac{dx}{dt} = 50$

Want $\frac{dV}{dt}$

$$V = x^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}[x^3]$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

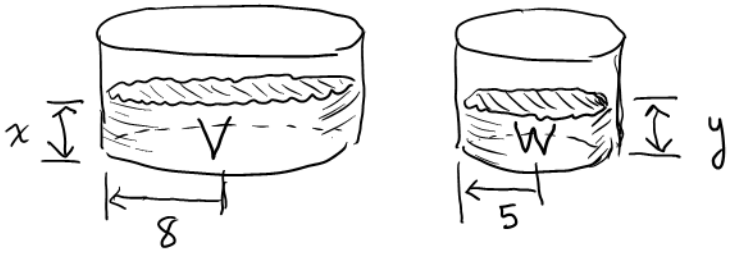
$$\frac{dV}{dt} = 3x^2 \, 2$$

$$\frac{dV}{dt} = 6x^2 \text{ cubic cm/sec}$$

Answer: When $x = 50$ cm, volume is changing at rate

$$\frac{dV}{dt} = 6 \cdot 50^2 = 6 \cdot 2500 = 15000 \text{ cubic cm/sec}$$

(44) Two pools are filling at the same rate



If their volumes are V and W, this means $\frac{dV}{dt} = \frac{dW}{dt}$. Call their water levels x and y as illustrated. It is known that the level of the smaller pool increases at a rate of 0.5 m/min, that is $\frac{dy}{dt} = 0.5$. How fast is the level of the larger pool increasing? That is, what is $\frac{dx}{dt}$?

$$V = \pi 8^2 x \qquad W = \pi 5^2 y$$

$$V = 64\pi x \qquad W = 25\pi y$$

$$\frac{d}{dt}[V] = \frac{d}{dt}[64\pi x] \qquad \frac{d}{dt}[W] = \frac{d}{dt}[25\pi y]$$

$$\frac{dV}{dt} = 64\pi \frac{dx}{dt} \qquad \frac{dW}{dt} = 25\pi \frac{dy}{dt}$$

But since we know $\frac{dV}{dt} = \frac{dW}{dt}$, the above gives

$$64\pi \frac{dx}{dt} = 25\pi \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{25\pi}{64\pi} \frac{dy}{dt} = \frac{25}{64} \cdot (0.5) = \frac{25}{128} = 0.1953125 \text{ m/min}$$