A parameter $\sigma$ is called \textit{submultiplicative} on a graph product $\otimes$ if $\sigma(G \otimes H) \leq \sigma(G)\sigma(H)$ for all graphs $G, H$. A parameter $\sigma$ is called \textit{supermultiplicative} on a graph product $\otimes$ if $\sigma(G \otimes H) \geq \sigma(G)\sigma(H)$ for all graphs $G, H$. A subset of vertices $S \subseteq V(G)$ is \textit{dominating} if every vertex in $V(G)$ is either in $S$ or adjacent to a vertex in $S$. The cardinality of the smallest dominating set is denoted by $\gamma(G)$, the domination number. The \textit{Cartesian product} of $G$ and $H$ is denoted by $G \square H$; the vertices are the ordered pairs $\{(x, y) | x \in V(G), y \in V(H)\}$. Two vertices $(u, v)$ and $(x, y)$ are adjacent in the Cartesian product if and only if one of the following is true: $u = x$ and $v$ is adjacent to $y$ in $H$; or $v = y$ and $u$ is adjacent to $x$ in $G$.

V.G. Vizing conjectured that for all graphs $G$ and $H$,

$$\gamma(G \square H) \geq \gamma(G)\gamma(H).$$

That is, Vizing conjectured that $\gamma$ was supermultiplicative on the Cartesian product. This conjecture has remained open since 1968. In contrast, it is not too hard to see why a variant of the domination number, the total domination number, is submultiplicative on the direct product.

A subset $S \subseteq V(G)$ is a \textit{2-packing} if the minimum distance between any distinct vertices $u, v$ of $S$ is at least three. The packing number $\rho(G)$ is the size of a largest 2-packing. It is known that Vizing’s conjecture will hold for every graph $H$, if $G$ satisfies at least one of the following: $\gamma(G) \leq 3$; $|\gamma(G) - \rho(G)| \leq 1$; $G$ is chordal; $G$ is the a spanning subgraph of $K$ with $\gamma(G) = \gamma(K) = \chi(K)$. In this talk, we outline failed attempts to construct a counter-example to Vizing’s conjecture, and in the process establish a lower bound on the number of vertices of a counterexample (should one exist).

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