1 Exercise 3

In these problems, assume a holonomic frame system and zero torsion so that
\[ S^\rho_{\mu\nu} = -2\Gamma^\rho_{[\mu\nu]} = \Gamma^\rho_{\nu\mu} - \Gamma^\rho_{\mu\nu} = 0 \]

1.1 Problem 3.1

The initial value structure of Maxwell’s Field Equations of Electromagnetism may be understood from the identity
\[ [g^{\mu\nu}g^{\sigma\rho}F_{\mu\sigma;\rho};\nu] = 0 \]
for any antisymmetric tensor field \( F \). Show that this expression is indeed an identity for fields in flat Minkowski Spacetime.

Answer 3.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.2 Problem 3.2

Now prove the identity
\[ [g^{\mu\nu} g^{\rho\sigma} F_{\mu\sigma,\rho}],\nu = 0 \]
for any Riemannian Manifold (metric-compatible connection with zero torsion). Problem 1 should show you how to start.

Answer 3.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.3 Problem 3.3

Start with the ADM decomposition of the spacetime metric tensor components in the form

\[
\begin{pmatrix}
N_r N^r - N^2 & N_1 & N_2 & N_3 \\
N_1 & h_{11} & h_{12} & h_{13} \\
N_2 & h_{21} & h_{22} & h_{23} \\
N_3 & h_{31} & h_{32} & h_{33}
\end{pmatrix}
\]

where

\[N^i = h^{ij} N_j\]

and

\[h^{ik} h_{kj} = \delta^i_j\]

and show that the inverse spacetime metric tensor components are then given by

\[
[g^{-1}] = \frac{1}{(N)^2} \begin{pmatrix}
-1 & N^1 & N^2 & N^3 \\
N^1 & (N)^2 h_{11} - N^1 N^1 & (N)^2 h_{12} - N^1 N^2 & (N)^2 h_{13} - N^1 N^3 \\
N^2 & (N)^2 h_{21} - N^2 N^1 & (N)^2 h_{22} - N^2 N^2 & (N)^2 h_{23} - N^2 N^3 \\
N^3 & (N)^2 h_{31} - N^3 N^1 & (N)^2 h_{32} - N^3 N^2 & (N)^2 h_{33} - N^3 N^3
\end{pmatrix}
\]

Note that Scientific Notebook will not cooperate in this case, because it will insist that all of the superscript indexes are powers.

**Answer 3.3**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.