1 Exercise 2

In the following problems, define the frame-field

\[ e_1 = \frac{\partial}{\partial r} \]
\[ e_2 = \frac{1}{r} \frac{\partial}{\partial \varphi} \]

where \( r, \varphi \) are coordinates on a manifold and use connection coefficients

\[
\begin{bmatrix}
\Gamma^1_{11} & \Gamma^1_{21} \\
\Gamma^2_{11} & \Gamma^2_{21}
\end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}
\]
\[
\begin{bmatrix}
\Gamma^1_{12} & \Gamma^1_{22} \\
\Gamma^2_{12} & \Gamma^2_{22}
\end{bmatrix} = \begin{bmatrix} \frac{1}{r} & -\frac{1}{r} \\
0 & 0
\end{bmatrix}
\]

Define a vector field

\[ v = -\frac{1}{1+r} e_1 + e_2 \]

a form field

\[ \alpha = \cos \varphi dr + r \sin \varphi d\varphi \]

and a tensor field

\[ T = r^2 \cos^2 \varphi e_1 \otimes dr - r^2 \sin^2 \varphi e_2 \otimes d\varphi \]

1.1 Problem 2.1

Calculate the derivatives \( D_v \alpha \) and \( D_v T \)

Answer 2.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.2 Problem 2.2

Define a vector-valued tensor field $S$ by

$$\gamma \cdot S(a) = T(\gamma, a)$$

for any one-form field $\gamma$ and vector field $a$. The quantity $\nabla_v S(a)$ is potentially ambiguous. Calculate the two things that it could mean, depending on where parentheses are placed:

$$(\nabla_v S)(a) \quad \text{and} \quad \nabla_v (S(a))$$

Answer 2.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.3 Problem 2.3

Find the pair of differential equations for $r(t), \varphi(t)$ that characterize a geodesic of this connection.

Answer 2.3

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.