1 Exercise 1

In the following problems, define the basis vectors

\[ e_1 = \frac{\partial}{\partial r} \]
\[ e_2 = \frac{1}{r} \frac{\partial}{\partial \varphi} \]

where \( r, \varphi \) are coordinates on a manifold and use connection coefficients

\[ [\Gamma]_1 = \begin{bmatrix} \Gamma_{11}^1 & \Gamma_{12}^1 \\ \Gamma_{21}^1 & \Gamma_{22}^1 \end{bmatrix} = [0] \]
\[ [\Gamma]_2 = \begin{bmatrix} \Gamma_{11}^2 & \Gamma_{12}^2 \\ \Gamma_{21}^2 & \Gamma_{22}^2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{r} \\ \frac{1}{r} & 0 \end{bmatrix} \]

Define a function

\[ f = r^2 \cos 2\varphi. \]

and vector fields

\[ u = \frac{1}{1 + r} e_1 + e_2 \]
\[ v = -\frac{1}{1 + r} e_1 + e_2 \]
\[ w = \cos \varphi e_1 + \sin \varphi e_2 \]

1.1 Problem 1.1

Calculate the derivatives \( \nabla_r f \) and \( \nabla_r w \).

Answer 1.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.2 Problem 1.2

Calculate the second derivatives $\nabla_u \nabla_v f$ and $D_u D_v f$.

**Answer 1.2**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
1.3 Problem 1.3

Calculate the second derivatives $\nabla_u \nabla_v w$ and $D_u D_v w$.

Answer 1.3

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.