PHYS691 Final Exam

Attempt each of the following problems. Attach the resulting file to an email to rhgowdy@vcu.edu.

Due date: Thursday, May 12, 2005.

1 Problem 1: Sound Waves

Use the stress-energy tensor conservation laws to find the speed of sound waves (as a fraction of the speed of light) in a medium that obeys an equation of state of the form

$$p = f(\rho)$$

Do the calculation for an arbitrary curved spacetime.

2 Problem 2: Bosons in Curved Spacetime

In Special Relativity, the wave function for a spin-zero massive particle obeys the Klein Gordon Equation

$$-\frac{\partial^2\psi}{\partial t^2} + \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = m^2\psi$$

a) Suppose that such a particle is moving through a curved spacetime and use minimal coupling to find a candidate for its wave equation.

b) Write out the candidate equation in detail for the case of a particle moving along the z-axis (so that $\frac{\partial\psi}{\partial x}$ and $\frac{\partial\psi}{\partial y}$ are zero).

3 Problem 3: Soap Films (Problem of Plateau)

A soap-film suspended on a wire frame with no air trapped anywhere will try to minimize its total surface area because of surface tension.

a) Represent such a film in parametric form in Cartesian coordinates.

b) Find the differential equations that are obeyed by the functions in this description of a soap film.

c) Find the condition(s) satisfied by the second fundamental form of such a soap film.

4 Problem 4: Gravitational Wave Sources

a) Use the results found in class, but ignore polarization effects and derive an approximate relationship between detector strain, source luminosity, source distance, and source frequency. For this part, just leave everything in Planck units.

b) For this part, you will need to look up some constants and conversions. Find the greatest distance (in light years) that a gravitational wave detector with a strain sensitivity of 10^{-18} could respond to an event that dumps one full solar mass of energy into a one second pulse of gravitational waves at an angular frequency of a kiloHertz.

5 Problem 5: Lapse and Shift

Find the lapse and shift functions that correspond to the spacetime metric tensor

$$ds^2 = -\left(1 - 2m/r\right) dt^2 + 2vP^{-1/2} dt dr + r^4 P^{-1} dr^2 + r^2 d\Omega^2$$

where v, P are polynomials

$$P = v^2 + \left(1 - 2m/r\right) r^4$$

$$v = Kr^3/3 - H$$

By the way, this is the metric of a black hole of mass m in peculiar coordinates.

6 Problem 6: Initial Data

Suppose that you wish to set up time-symmetric initial data for two black holes of identical mass separated by about ten Schwarzschild radii. The data is to be set up on a Cartesian coordinate grid (x, y, z) with the holes on the z-axis.

For a single black hole the horizon corresponds to the minimal area r=constant surface at the instant of time symmetry. Assume that this relationship is approximately true for these interacting black holes so that their minimal area surfaces (now somewhat distorted) correspond to their horizons and give the spacetime Cartesian metric tensor components as functions of the coordinates (x, y, z).

7 Problem 7: Isometries

Use the procedures that we applied to the case of static spherical symmetry and construct a simple form for the metric of a static, cylindrically symmetric spacetime. Take the coordinates to be (t, r, z, θ). In this case, the Killing vectors are $\frac{\partial}{\partial t}, \frac{\partial}{\partial z}, \frac{\partial}{\partial \theta}$. Be sure to justify each specialization.

8 Problem 8:

For this problem, you will have to draw some pictures.

Use a Kruskal Diagram to show the geometry near the surface of a star that is collapsing to a black hole. An observer is standing off from the collapse at a constant luminosity distance of $r = 3m$.

a) What happens to the initial $r = 0$ singularity of the Kruskal metric in this picture?

b) Suppose that a clock is on the surface of the star and is sending out light signals are regular intervals. Use the Kruskal Diagram to explain what the $r = 3m$ observer will see in terms of the time, t for which the external geometry is static.