Hour Exam No.1

Please attempt all of the following problems before the due date. All problems count the same even though some are more complex than others.

Problem 1

At Minkowski coordinate time, t, an object is located at the Minkowski position coordinates

$$x(t) = \sqrt{t^2 + 1}; \quad y(t) = z(t) = 0.$$

Using $c = 1$ units, find

a. the object's ordinary Newtonian velocity vector components.

Answer 1a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. the components of the object's four-velocity vector.

Answer 1b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 2

Consider a two-dimensional spacetime manifold where we are using the coordinates t, x to locate events and the corresponding holonomic basis vectors

$$\partial_t = \frac{\partial}{\partial t}, \quad \partial_x = \frac{\partial}{\partial x}$$

to span each tangent space. A different coordinate system t', x' also locates events in this spacetime where

$$t' = t; \quad x' = x - \sqrt{t^2 + 1},$$

and the corresponding holonomic basis vectors

$$\partial_{t'} = \frac{\partial}{\partial t'}, \quad \partial_{x'} = \frac{\partial}{\partial x'}$$

also span each tangent space.

a. Notice that $\partial_{t'}$ is not equal to ∂_t even though $t' = t$. Express $\partial_{t'}$ in terms of ∂_t and ∂_x.

Answer 2a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Express $\partial_{x'}$ in terms of ∂_t and ∂_x.

Answer 2b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

c. Quick answer: What differential forms correspond to the basis dual to the basis vectors ∂_t and ∂_x?

Answer 2c

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 3

Suppose that the metric tensor on a spacetime has the form

$$g = -dt \otimes dt + dx \otimes dx$$

and you decide to use the "null" coordinates

$$\begin{aligned} u^1 &= t + x \\ u^2 &= t - x \end{aligned}$$

and the corresponding "null basis" vectors

$$e_1 = \frac{\partial}{\partial u^1}, \quad e_2 = \frac{\partial}{\partial u^2}.$$

a. Express the null basis vectors in terms of the basis vectors ∂_t and ∂_x that go with the coordinates x, t.

Answer 3a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the metric components g_{11}, g_{12}, g_{22} in the null basis.

Answer 3b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 4

An electromagnetic field two-form is given by

$$f = r^{-2} dt \wedge dr$$

where t is the usual Minkowski time function and $r = \sqrt{x^2 + y^2 + z^2}$ is a radius coordinate. In the following, use the basis vectors

$$\partial_0 = \frac{\partial}{\partial t}, \quad \partial_1 = \frac{\partial}{\partial r}$$

and their dual basis forms and assume $c = 1$ units.

a. Find the components $F_{00}, F_{01}, F_{10}, F_{11}$, of f.

Answer 4a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the components $F^1{}_0$ and $F^0{}_1$ of the related tensor.

Answer 4b

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c. Identify the sort of object that would make this electromagnetic field.

Answer 4c

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Problem 5

Suppose that the metric tensor on a spacetime has the form

$$g = -dt \otimes dt + dx \otimes dx$$

and you decide to use the "null" coordinates

$$\begin{aligned} u^1 &= t + x \\ u^2 &= t - x \end{aligned}$$

and the corresponding "null basis" vectors

$$e_1 = \frac{\partial}{\partial u^1}, \quad e_2 = \frac{\partial}{\partial u^2}.$$

a. Express the null basis vectors in terms of the basis vectors ∂_t and ∂_x that go with the coordinates x, t.

Answer 5a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the metric components g_{11}, g_{12}, g_{22} in the null basis.

Answer 5b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 6

A set of observers are sitting on a flat turntable that is rotating with angular velocity ω. Each observer is located at a fixed pair of Cartesian coordinates x', y' that rotate with the disk. In terms of the non-rotating Minkowski coordinates t, x, y the position of an given observer at x', y' is

$$\begin{aligned} x &= x' \cos \omega t - y' \sin \omega t \\ y &= x' \sin \omega t + y' \cos \omega t \end{aligned}$$

Using $c = 1$ units,

a. find the four-velocity vector u of the observer at

$$\begin{aligned} x' &= r \\ y' &= 0 \end{aligned}$$

in terms of the non-rotating Minkowski basis vectors $\partial_t, \partial_x, \partial_y$

Answer 6a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. The co-rotating coordinate system consists of $t' = t, x', y'$. Note that it is still the same t. Express the co-rotating holonomic basis vectors $\partial_{x'}$ and $\partial_{y'}$at

$$\begin{aligned} x' &= r \\ y' &= 0 \end{aligned}$$

in terms of the non-rotating Minkowski basis vectors $\partial_t, \partial_x, \partial_y$

Answer 6b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

C. Suppose that the observer at

$$\begin{aligned} x' &= r \\ y' &= 0 \end{aligned}$$

uses the basis vectors

$$\begin{aligned} e_0 &= u \\ e_1 &= \partial_{x'} \\ e_2 &= \partial_{y'} \end{aligned}$$

Find the components of the metric tensor in this basis.

Answer 6c

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.