Hour Exam No.1

Please attempt all of the following problems before the due date. All problems count the same even though some are more complex than others.

Problem 1

At Minkowski coordinate time, $t$, an object is located at the Minkowski position coordinates

$$x(t) = \sqrt{t^2 + 1}; \quad y(t) = z(t) = 0.$$ 

Using $c = 1$ units, find

a. the object’s ordinary Newtonian velocity vector components.

Answer 1a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. the components of the object’s four-velocity vector.

Answer 1b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 2

Consider a two-dimensional spacetime manifold where we are using the coordinates \( t, x \) to locate events and the corresponding holonomic basis vectors

\[
\partial_t = \frac{\partial}{\partial t}, \quad \partial_x = \frac{\partial}{\partial x}
\]

to span each tangent space. A different coordinate system \( t', x' \) also locates events in this spacetime where

\[
t' = t; \quad x' = x - \sqrt{t^2 + 1},
\]

and the corresponding holonomic basis vectors

\[
\partial_{t'} = \frac{\partial}{\partial t'}, \quad \partial_{x'} = \frac{\partial}{\partial x'}
\]

also span each tangent space.

a. Notice that \( \partial_{t'} \) is not equal to \( \partial_t \) even though \( t' = t \). Express \( \partial_{t'} \) in terms of \( \partial_t \) and \( \partial_x \).

Answer 2a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Express \( \partial_{x'} \) in terms of \( \partial_t \) and \( \partial_x \).

Answer 2b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

C. Quick answer: What differential forms correspond to the basis dual to the basis vectors \( \partial_t \) and \( \partial_x \)?

Answer 2c

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 3

Suppose that the metric tensor on a spacetime has the form

\[ g = -dt \otimes dt + dx \otimes dx \]

and you decide to use the “null” coordinates

\[ u^1 = t + x \]
\[ u^2 = t - x \]

and the corresponding “null basis” vectors

\[ e_1 = \frac{\partial}{\partial u^1}, \quad e_2 = \frac{\partial}{\partial u^2}. \]

a. Express the null basis vectors in terms of the basis vectors \( \partial_t \) and \( \partial_x \) that go with the coordinates \( x, t \).

Answer 3a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the metric components \( g_{11}, g_{12}, g_{22} \) in the null basis.

Answer 3b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 4

An electromagnetic field two-form is given by

\[ f = r^{-2} dt \wedge dr \]

where \( t \) is the usual Minkowski time function and \( r = \sqrt{x^2 + y^2 + z^2} \) is a radius coordinate. In the following, use the basis vectors

\[ \partial_0 = \frac{\partial}{\partial t}, \quad \partial_1 = \frac{\partial}{\partial r} \]

and their dual basis forms and assume \( c = 1 \) units.

\[ \text{a. Find the components } F_{00}, F_{01}, F_{10}, F_{11}, \text{ of } f. \]

Answer 4a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

\[ \text{b. Find the components } F_{10}^1 \text{ and } F_{01}^0 \text{ of the related tensor.} \]

Answer 4b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

\[ \text{c. Identify the sort of object that would make this electromagnetic field.} \]

Answer 4c

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 5

Suppose that the metric tensor on a spacetime has the form
\[ g = -dt \otimes dt + dx \otimes dx \]
and you decide to use the “null” coordinates
\[ u^1 = t + x, \quad u^2 = t - x \]
and the corresponding “null basis” vectors
\[ e_1 = \frac{\partial}{\partial u^1}, \quad e_2 = \frac{\partial}{\partial u^2}. \]

a. Express the null basis vectors in terms of the basis vectors \( \partial_t \) and \( \partial_x \) that go with the coordinates \( x, t \).

Answer 5a

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. Find the metric components \( g_{11}, g_{12}, g_{22} \) in the null basis.

Answer 5b

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
Problem 6

A set of observers are sitting on a flat turntable that is rotating with angular velocity $\omega$. Each observer is located at a fixed pair of Cartesian coordinates $x', y'$ that rotate with the disk. In terms of the non-rotating Minkowski coordinates $t, x, y$ the position of an given observer at $x', y'$ is

\[
\begin{align*}
x &= x' \cos \omega t - y' \sin \omega t \\
y &= x' \sin \omega t + y' \cos \omega t
\end{align*}
\]

Using $c = 1$ units,

a. find the four-velocity vector $u$ of the observer at

\[
\begin{align*}
x' &= r \\
y' &= 0
\end{align*}
\]

in terms of the non-rotating Minkowski basis vectors $\partial_t, \partial_x, \partial_y$

**Answer 6a**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

b. The co-rotating coordinate system consists of $t', x', y'$. Note that it is still the same $t$. Express the co-rotating holonomic basis vectors $\partial_{x'}$ and $\partial_{y'}$ at

\[
\begin{align*}
x' &= r \\
y' &= 0
\end{align*}
\]

in terms of the non-rotating Minkowski basis vectors $\partial_t, \partial_x, \partial_y$

**Answer 6b**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.
C. Suppose that the observer at

\[ x' = r \]
\[ y' = 0 \]

uses the basis vectors

\[ e_0 = u \]
\[ e_1 = \partial_{x'} \]
\[ e_2 = \partial_{y'} \]

Find the components of the metric tensor in this basis.

**Answer 6c**

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.