Exercise 06

Please attempt all of the following problems before the due date. Your grade on this assignment will be calculated from the best two answers.

Problem 06.1

The kinetic energy of a moving object (in three dimensional space) takes the general form

\[ K.E. = \frac{1}{2} m g(v, v) \]

where \( v \) is the object’s velocity (tangent vector to the curve parameterized by time) and \( g \) is a second rank covariant tensor. Ordinarily we take \( g \) to be a symmetric tensor. Einstein considered theories in which \( g \) is replaced by a tensor \( g + a \) where \( a \) is antisymmetric. Show that the antisymmetric term does not affect the kinetic energy expression.

Answer 06.1

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 06.2

A two dimensional coordinate system is transformed from coordinates \( x, y \) to new coordinates \( X(x, y), Y(x, y) \). Express the new basis forms \( dX \) and \( dY \) in terms of the old basis forms \( dx \) and \( dy \) and then express the wedge product \( dX \wedge dY \) in terms of the wedge product \( dx \wedge dy \).

Answer 06.2

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.

Problem 06.3

The three dimensional vector cross product can be defined in tensor index notation as

\[ (B \times C)^i = \varepsilon^{ijk} B_j C_k \]

where \( \varepsilon^{ijk} \) is the totally antisymmetric Levi-Civita symbol. Notice that the triple product \( A \cdot (B \times C) \) is then just

\[ A \cdot (B \times C) = A_i (B \times C)^i \]
Use the connection between the Levi-Civita symbol and the determinant of a matrix to derive the identity

\[ A \cdot (B \times C) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} . \]

Answer 06.3

Put all of your calculations here. When you have completed all of the problems, wrap the resulting file and e-mail it to me at rgowdy@saturn.vcu.edu.