Expectations Anchoring in Inflation Targeting Regimes

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Abstract

Central banks adopt an inflation targeting policy with a goal to anchor inflation expectations. We argue that the expectations anchoring test developed in the context of the Krugman (1991) exchange rate targeting model is well-suited for inflation targeting applications. The test quantifies nonlinearity between realized and expected inflation for very high and very low inflation levels. It does not require comparison with the control group of non-targeting countries, avoiding critique of the benchmark approach. We test inflation targeting in Australia, Canada, New Zealand, Sweden, the United Kingdom and find weak support for expectations anchoring.

Keywords: monetary policy, inflation, exchange rate, target zone model, expectations anchoring

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1 Introduction

High inflation fuels future expectations of even higher inflation. To anchor inflation expectations, a central
don bank adopts the inflation targeting policy by making a promise to contain inflation inside announced bounds.
With a trustworthy central bank, there is no reason to expect high inflation in the future, even if inflation
is high today. Similarly, there is no reason to expect deflation in the future, even if inflation is very low
today. As a result, for very high and very low levels of inflation, inflation targeting weakens the link between
realized and expected inflation.

Inflation targeting is gaining popularity despite an ongoing empirical debate on whether this policy
is successful at anchoring inflation expectations.\(^1\) Levin, Natalucci, and Piger (2004), Gurkaynak, Levin,
and Swanson (2006) provide evidence favoring the view that inflation targeting plays a significant role
in anchoring inflation expectations. Other studies conclude that the policy does not change expectations
the first few years after adopting inflation targeting, expected inflation exceeds realized inflation. Johnson
(2002) demonstrates that the policy does not reduce dispersion of inflation forecasts. Ball and Sheridan
(2005) show that the inflation targeting policy does not change significantly the relation between one-year
ahead inflation expectations and realized inflation.

The benchmark test of expectation anchoring, used by Ball and Sheridan (2005) and Levin, Natalucci, and
Piger (2004), regresses inflation expectations on realized inflation in two groups of countries: the targeting
group and the non-targeting group. A significantly smaller coefficient on realized inflation in the targeting
group is interpreted as evidence of the weak link between realized and expected inflation. Even though the
choice of the inflation targeting countries is obvious, Gertler (2005) and Uhlig (2004) point out that the
choice of non-inflation targeting countries is hard to defend. Some central banks, in particular the central
banks of Germany and the U.S., intervene to lower inflation rates, as inflation targeting countries do. Also,
European countries had to satisfy inflation criteria to join the European monetary union, even though they
did not adopt explicit inflation targeting.

We propose an alternative expectations anchoring test that is based on the Krugman (1991) model.
Following Amano, Black and Kasumovich (1997) and Tetlow (2000), we argue that the Krugman model,
developed in the context of exchange rate targeting, is applicable to inflation targeting. Our test does
not require comparison with a control group of non-targeting countries, avoiding critique of the benchmark
approach. In addition to the test, our approach allows us to measure the degree of expectation anchoring
and the strength of anticipated central bank interventions. We test inflation targeting in Australia, Canada,
New Zealand, Sweden, the United Kingdom and find weak support for expectations anchoring.

\(^1\)Seventeen central banks (Australia, Brazil, Canada, Chile, the Czech Republic, Finland, Hungary, Israel, Korea, New
Zealand, Poland, South Africa, Spain, Sweden, Switzerland, Thailand, and the United Kingdom) have adopted explicit inflation
targeting (IT) over the past two decades.
Amano, Black and Kasumovich (1997) and Tetlow (2000) conclude that, after allowing for inflation to break the limits of target zone, the Krugman model should be useful for an empirical evaluation of inflation targeting regimes. Even though Krugman developed his model in a context of exchange rate targeting, the two policies are similar. Exchange rate and inflation targeting policies have the same goal (to make the target variable more predictable), use interventions to achieve this goal, and allow infrequent breaks of the target zone. The main difference is that interventions affect an exchange rate faster than inflation, but still not fast enough to keep an exchange rate inside the target zone at all times. Our empirical model describes the behavior of the target both inside and outside the target zone allowing for the target to break the bounds.

When applied to inflation targeting, the Krugman model predicts that the link between realized and expected inflation is nonlinear: The link weakens as inflation approaches the boundaries. For example, when inflation is high, agents in economy anticipate central bank intervention. The anticipated intervention makes the probability of higher inflation in the future small compared to the probability of lower inflation. Even though current inflation is high, it does not translate into high expected inflation, weakening the link between realized and expected inflation close to the boundary. This nonlinearity is a testable implication of the Krugman model, applied to inflation targeting.

We test for the nonlinear dependence of expected and realized inflation using the Forbes and Kofman (2000) version of the Krugman model. The Forbes and Kofman model retains the nonlinear dependence while allowing for the target variable to leave the target zone. In addition, the alternative where expectations depend linearly on realized inflation, resulting from a failure of inflation targeting to anchor expectations, is nested in this model. The nested test is preferable to the Ruge-Murcia (2000) test where linear and non-linear dependence are non-nested, which results in complicated asymptotic tests with unclear small sample properties. The complications make Ruge-Murcia conclude that the evidence of nonlinearity “should be best regarded only as indicative”.

The rest of the paper is organized as follows: In Section 2, we argue that the Krugman model is applicable in the context of inflation targeting. In Section 3, we discuss the model setup, while Section 4 discusses estimation. Details on the data are contained in Section 5, and empirical results are gathered in Section 6. Concluding remarks are given in Section 7. Appendix illustrates the methodology for the daily Norwegian exchange rate index.

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2Krugman assumes that interventions occur when the target variable is at the upper or lower bound. Delgado and Dumas (1991) extend the result to interventions that occur inside the target zone.

3Alternative empirical approaches in the context of exchange rate targeting can be found in Lundberg and Terasvirta (2006), Li (1999), Pesaran and Ruge-Murcia (1999) among many others.
2 Inflation Versus Exchange Rate Targeting

Amano, Black and Kasumovich (1997), and Tetlow (2000), consider applicability of the Krugman model to inflation targeting. Both papers conclude that even though the monetary authority is not able to control inflation instantaneously, the model that incorporates the ability of the target variable to break the limits of target zone should still retain nonlinearity of expectations and thus should be useful for an empirical evaluation of inflation targeting regimes.

Exchange rate and inflation targeting policies have a similar goal, use interventions to achieve it, and have similar flexibility in implementing the policy. The goal of both policies is to make a target variable – inflation or exchange rate – more predictable. Both policies achieve predictability by promising to keep the target variable inside the specified bounds. When the target variable is close to the bounds, a central bank intervenes to bring it back to the center of the target zone. Finally, both policies are flexible – a central bank allows the target to move outside the zone or even can change the bounds of the zone.

Though there are important similarities between exchange and inflation targeting regimes, there are also two differences, which we address in our empirical model. The obvious difference is the target variable. For an exchange rate policy, the target variable is the price level of one currency in terms of the other. Most price levels, including the exchange rates, are unit root processes. To address nonstationarity, empirical models of exchange rate are written in first differences. For inflation targeting policy, the target variable is the change in price level. The target variable is already a first difference of aggregate price level. Inflation is persistent but stationary. Stationarity implies that an empirical model does not require difference transformation and can be estimated in levels. To account for inflation persistence, we introduce lagged inflation. Our empirical model nests the model in differences when we restrict inflation autocorrelation to be equal to one.

The more important difference is that the period between intervention and its effect is much longer for inflation targeting. Inflation targeting intervention changes an interest rate. A change in interest rate affects inflation with a delay that can be as long as one year. Exchange rate policy intervention is a change in supply of foreign currency. A change in supply affects exchange rate with a short or no delay. Still there are instances when the effect of interventions is not fast enough to keep an exchange rate inside the target zone at all times. Our empirical model allows for a delay between a policy intervention and the target variable response by describing the behavior of the target variable both inside and outside the target zone. Also, different data frequency mitigates a longer intervention delay for inflation targeting. Exchange rate observations have a daily frequency, while the frequency of inflation observations is monthly or even quarterly. One period delay for exchange rate means a single day, while for inflation it could be a month or even a quarter.
3 Target Zone Model

Our model of the target variable is an autoregressive time series. Inflation lags explain significant part of inflation variation in many empirical evaluation of structural models. We build on the Forbes and Kofman (2000) target zone model, which modifies and extends the Bekaert and Gray (1998) model.

Let $U_t$ be an upper bound, $C_t$ be a center, and $L_t$ be a lower bound of a target zone. Subscript $t$ indicates that a monetary policy can change the design of the target zone by changing the bounds or the center of the target zone. We define a general target variable $\tau_t$ by normalizing inflation or exchange rate,

$$\tau_t = \frac{2(x_t - C_t)}{(U_t - L_t)}$$

where $x_t$ is either exchange rate or inflation. By normalizing a target variable, we also normalize the target zone: the center is equal to zero, the lower bound is equal to $-1$, and the upper bound is equal to $1$. The target model then becomes:

$$\tau_t = c + a(L)\tau_t + \epsilon_t, \quad t = 1, 2, ..., T \quad (1)$$

where $a(L) = a_1L + a_2L^2 + ...$ is a polynomial lag operator and $c$ is a constant. The goal of a targeting policy is to keep the target variable $\tau_t$ inside the target zone $[-1, 1]$. Anticipated central bank interventions in our model modify the error term. If agents in economy do not anticipate interventions, the error term in equation (1) has the Normal distribution. The anticipated interventions are incorporated in the model by restricting the values that error term $\epsilon_t$ can take.

3.1 Perfectly Credible Target Zone

We call the target zone perfectly credible when agents in economy believe that the central bank keeps the target variable within the bounds $-1 \leq \tau_t \leq 1$ at all times, $t = 1, 2, ..., T$. This belief is incorporated via the error term, error can never be too large or too small to move the target outside the target zone. Such error has the truncated Normal distribution with the probability density function

$$p(\epsilon_t) = \frac{\frac{1}{\sigma} \phi \left( \frac{\epsilon_t}{\sigma} \right)}{\Phi \left( \frac{U'_t}{\sigma} \right) - \Phi \left( \frac{L'_t}{\sigma} \right)} 1_{[L'_t, U'_t]}(\epsilon_t),$$

where $1_A(x)$ is an indicator function which takes on a value 1 if $x \in A$ and zero otherwise, $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard Normal density and cumulative distribution functions, $\sigma$ is a standard deviation of the error term, $U'_t = 1 - c - a(L)\tau_t$ and $L'_t = -1 - c - a(L)\tau_t$ are the minimum and maximum values that the error

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4See Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003) for the representative empirical evaluations of structural models. See Fuhrer(2005), Rudd and Whelan (2005), Korenok, Radchenko and Swanson (2006) for a detailed discussion on importance of lags in structural models.
term can take. The target variable is at the upper bound when the error term is equal to $U_t^\epsilon$. The target is at the lower bound when the error term is equal to $L_t^\epsilon$.

The expectation of the error term conditional on information up to period $t$ is:

$$E(\epsilon_t | I_{t-1}) = \sigma \frac{\phi \left( \frac{L_t^\epsilon}{\sigma} \right) - \phi \left( \frac{U_t^\epsilon}{\sigma} \right)}{\Phi \left( \frac{U_t^\epsilon}{\sigma} \right) - \Phi \left( \frac{L_t^\epsilon}{\sigma} \right)}.$$ (2)

Expectation is a function of $\sigma$, $L_t^\epsilon$ and $U_t^\epsilon$. The values of $L_t^\epsilon$ and $U_t^\epsilon$ in turn are functions of the position of the target variable in the zone $c + a(L)\tau_t$. In perfectly credible target zone, the Krugman model, applied to inflation targeting, predicts that expected inflation depends non-linearly on realized inflation, or more generally, on the position in the zone. In our model, the only source of non-linearity is expectation of the error term, which depends non-linearly on the position in the zone.

The nonlinear dependence, illustrated by the solid line in Figure 1, has the S-shape form: expectation is zero around the center, negative at the upper bound, and positive at the lower bound. Without loss of generality, assume the target variable follows a pure random walk, $a(L) = L$ and $c = 0$, then the position in the zone refers to the value of the target variable in the last period $\tau_{t-1}$, for example, realized inflation. When realized inflation is at the center of the zone $\tau_{t-1} = 0$, agents in economy do not anticipate central bank interventions, and the distribution of error term is symmetric with the support $[L_t = -1, U_t = 1]$ and zero mean. When realized inflation is high, close to the upper bound $\tau_{t-1} = 1$, agents anticipate central bank to intervene, bringing inflation down. As a result, the right tail of the error distribution is truncated more with the support $[-2, 0]$. This distribution is skewed to the left with a negative mean, which reflects agents’ anticipation of future central bank interventions. Similarly, when realized inflation is near the lower bound, the error distribution is skewed to the right with positive mean, reflecting agents’ anticipation of central bank interventions.

### 3.2 Non-Perfectly Credible Target Zone

We call a target zone non-perfectly credible when agents in economy anticipate target variable to move outside the zone because of a delay between a central bank intervention and a target variable response. In the Forbes and Kofman target zone model, the error term has the probability density function:

$$f(\epsilon_t) = \frac{\frac{1}{\sigma} \phi \left( \frac{U_t^\epsilon}{\sigma} \right) (1 - \alpha_t)}{\Phi \left( \frac{U_t^\epsilon}{\sigma} \right) - \Phi \left( \frac{L_t^\epsilon}{\sigma} \right)} 1_{L_t^\epsilon, U_t^\epsilon}(\epsilon_t) + \frac{\frac{1}{\sigma} \phi \left( \frac{U_t^\epsilon}{\sigma} \right) \alpha_t}{\Phi \left( \frac{U_t^\epsilon}{\sigma} \right) + 1 - \Phi \left( \frac{U_t^\epsilon}{\sigma} \right)} \{1_{(-\infty, L_t^\epsilon)}(\epsilon_t) + 1_{(U_t^\epsilon, \infty)}(\epsilon_t)\}$$ (3)
where agents anticipate the target variable to move outside the zone with probability $\alpha_t$. This probability is restricted by a maximum value $\alpha^*$:

$$\alpha_t = \min\{\alpha^*, \tilde{\alpha}_t\}. \quad(4)$$

where $\tilde{\alpha}_t = \Phi\left(\frac{L^*}{\sigma}\right) + 1 - \Phi\left(\frac{U^*}{\sigma}\right)$.

We interpret $\tilde{\alpha}_t$ as an unrestricted probability of the target variable to leave the zone, i.e., agents anticipate no central bank interventions; $\alpha_t = \min(\alpha^*, \tilde{\alpha}_t)$ as a restricted, adjusted by the anticipation of central bank interventions, probability of the target variable to leave the zone; and $\alpha^*$ as a restriction imposed by anticipated central bank interventions.

In our model agents incorporate anticipated central bank interventions restricting the distribution of the error term. A target variable is outside the zone when the error term is outside the interval $[L^*_t, U^*_t]$. Probability that the error term with the unrestricted Normal distribution leaves this interval equals $\Phi\left(\frac{L^*_t}{\sigma}\right) + 1 - \Phi\left(\frac{U^*_t}{\sigma}\right)$, which is a definition of $\tilde{\alpha}_t$.

When the position of the target is close to the center, the probability of breaking the zone without intervention $\tilde{\alpha}_t$ is small. Agents do not anticipate central bank interventions, leaving the probability of breaking the zone unrestricted $\alpha_t = \tilde{\alpha}_t$. But, as the position of the target variable in the zone moves closer to the bound, the probability of breaking the target zone increases. Anticipated interventions decrease the probability of moving outside the target zone from $\tilde{\alpha}_t$ to a constant $\alpha^*$. Interventions improve policy credibility by making the move outside the target zone less likely, thus decreasing $\alpha^*$. Figure 2 provides an example of $\tilde{\alpha}_t$ and $\alpha_t$ given $\alpha^* = 0.2$, $c = 0$ and $\sigma = 0.2$.

As in a non-perfectly credible target zone, the expected error term retains the nonlinearity, conditional on information up to period $t$:

$$E(\epsilon_t|I_{t-1}) = (1 - \alpha_t)\sigma \frac{\phi\left(\frac{L^*_t}{\sigma}\right) - \phi\left(\frac{U^*_t}{\sigma}\right)}{\Phi\left(\frac{U^*_t}{\sigma}\right) - \Phi\left(\frac{L^*_t}{\sigma}\right)} + \alpha_t\sigma \frac{\phi\left(\frac{U^*_t}{\sigma}\right) - \phi\left(\frac{L^*_t}{\sigma}\right)}{\Phi\left(\frac{L^*_t}{\sigma}\right) + 1 - \Phi\left(\frac{U^*_t}{\sigma}\right)}, \quad t = 1, 2, ..., T. \quad(5)$$

Close to the center of the target zone, the expected error is zero. Agents do not anticipate the central bank to intervene because the unrestricted probability of breaking the zone $\tilde{\alpha}_t$ is small. The unrestricted distribution is Normal with zero expected value. Close to the bounds of the zone, the expected error is non-zero. Agents anticipate central bank to intervene reducing the probability of breaking the zone from $\alpha_t$ to $\alpha^*$. Small $\alpha_t$ puts large weight on a first summand of the equation (5), which is identical to an expected error in the perfectly credible target zone model, and so is non-zero.

The dash-dotted line in Figure 1 illustrates the S-shape of the conditional expected error in the non-perfectly credible target zone for $\alpha^* = 0.2$ and $\sigma = 0.2$. The expected error of the non-perfectly credible
target zone is in between of the perfectly credible expected error, the solid line, and completely unrestricted expected error, which equals zero independent of the position in the zone. Compared to the solid line, the error term in the non-perfectly credible zone is zero over a wider range. As the central bank relaxes the restriction on a probability of leaving the zone and \( \alpha^* \) rises, the expected error term gets closer to a zero line.

### 3.3 Strength of Anticipated Policy Interventions

We interpret the distance between unrestricted \( \tilde{\alpha}_t \) and restricted \( \alpha_t \) probabilities of breaking the target zone as a measure of anticipated intervention strength. When a target variable is close to the center, both the strength of anticipated intervention and a difference between restricted and unrestricted probabilities are zero. As the target variable moves closer to the bounds, a stronger central bank intervention is needed to bring the target back to the center. At the same time, the unrestricted probability of leaving the zone increases. Because anticipated interventions restrict the probability to a constant \( \alpha^* \), higher unrestricted probability means larger distance between unrestricted and restricted probabilities. As a result, anticipated intervention strength can be measured by the distance between unrestricted and restricted probabilities.

As a measure of anticipated intervention strength, the distance between unrestricted and restricted probabilities is hard to interpret. How large is an anticipated intervention that decreases the probability of moving outside the zone from 20\% to 10\%? Is it equivalent to an anticipated intervention that decreases the probability from 30\% to 20\%? How does a decrease in probability translate into a change in the level of the target variable? To simplify interpretation, we translate the distance between the probabilities into the distance between positions of the target variable in the target zone. The distance between positions in the zone has the same unit of measurement as the target variable.

We translate the distance between the probabilities into the distance between positions in the zone, inverting the unrestricted probability function \( \tilde{\alpha}_t \) of position in the zone \( a(L)\tau_t \):

\[
\tilde{\alpha}_t = f(a(L)\tau_t) = \Phi \left( \frac{-1 - c - a(L)\tau_t}{\sigma} \right) + 1 - \Phi \left( \frac{1 - c - a(L)\tau_t}{\sigma} \right).
\]

The function is symmetric around zero, continuous and strictly monotonic on intervals \((-\infty, 0)\) and \((0, \infty)\), thus invertible on these intervals. There is a single corresponding value of position in the zone, \( f^{-1}(\tilde{\alpha}_t) \), for each value of unrestricted probability. Using the same function, we also find the position in the zone that corresponds to a value of restricted probability, \( f^{-1}(\alpha_t) \). The difference between the two positions is our measure of anticipated intervention strength.
4 Estimation Details

The model is estimated in Bayesian framework. We choose standard prior distributions for a linear regression model except for $\alpha^*$. Autoregressive parameters $a_1, \ldots, a_p$ have truncated Normal prior distribution centered at random walk specification with variance equal to one, where truncation is imposed to guarantee the stationarity of the target variable. The standard deviation of error term $\sigma$ has Inverted Gamma distribution with parameters $\nu = 1$ and $s = 0.005$. Prior distribution of $\alpha^*$ is Beta with parameters $\gamma$ and $\delta$. We set $\gamma = 1$, $\delta = 1.7$ which implies almost flat prior slightly favoring smaller values, the value of $\alpha^*$ less than 0.5, i.e. $p(\alpha^* < 0.1) = 0.16$ and $p(\alpha^* < 0.4) = 0.58$. Table 1 summarizes the prior distributions for the model parameters.

The likelihood of the model is given by:

$$L(c, a(L), \sigma, \alpha^*) \propto \sigma^{-(T-1)} \prod_{t=2}^{T} \phi \left( \frac{\tau_t - c - a(L)\tau_t}{\sigma} \right) \left[ \frac{(1 - \alpha_t)1_{[L^*_t, U^*_t]}(\tau_t - c - a(L)\tau_t)}{\Phi \left( \frac{L^*_t}{\sigma} \right) - \Phi \left( \frac{U^*_t}{\sigma} \right)} \right] + \frac{\alpha_t 1_{[-\infty, L^*_t]}(\tau_t - c - a(L)\tau_t)}{\Phi \left( \frac{L^*_t}{\sigma} \right) + 1 - \Phi \left( \frac{L^*_t}{\sigma} \right)}$$

(6)

In estimation, we use the Markov Chain Metropolis-Hastings algorithm. We draw parameters $a_1, \ldots, a_p$ and $\alpha^*$ using the griddy Gibbs sampling. The draw of $\sigma$ is done using the Metropolis-Hastings algorithm. In the implementation of the algorithm, we make 20,000 draws and discard the first 5,000 as a convergence region. The resulting number of draws used in the estimation of parameters is 15,000.

5 Data

All countries stabilize inflation in a medium run (one to two years interval). A medium run is justified by delays between the intervention and inflation response and by another goal of monetary policy to stabilize the fluctuations of real activity. A small increase in the interest rate decreases inflation, bringing it back inside the zone with a one- or two-year delay. A large increase in the interest rate, on the other hand, can decrease inflation immediately, but it negatively affects real economic activity and thus is undesirable.

charges. Finally, even though, Riksbank uses a standard CPI, it does not respond to short run fluctuations; Deputy Governor, Heikensten (1999) states “...certain types of transitory and sudden shocks are allowed to affect the CPI without prompting policy adjustments.”

Short run fluctuations are not only removed but also are averaged out. All countries effectively target a one-year average of monthly inflation. A target variable is defined as a year-over-year rate of change of price level – the difference between the current price level and the price level one year ago. Such difference is equivalent to an average of monthly inflation over one year:

\[ \pi_{t,13} = p_t - p_{t-13} = (p_t - p_{t-1}) + (p_{t-1} - p_{t-2}) + \ldots + (p_{t-12} - p_{t-13}) \]

\[ = \sum_{k=0}^{12} \Delta p_{t-k}, \]

(7)

where \( p_t \) is the current price level. In the empirical model, we take the same definition of inflation as monetary authorities.\(^5\)

Figure 3 summarizes the dynamics of target variables inside the target zones. The data span from December 1993 to June 2005 for Canada, from December 1994 to July 2005 for Sweden, and from October 1992 to October 2003 for the United Kingdom.\(^6\) All central banks allow inflation to move outside the target zone. In Australia, Sweden, and New Zealand, inflation stays outside the zone for extended periods of time. Target breaks emphasize the importance of modeling the behavior of inflation both inside and outside the target zone.

6   Empirical Findings

We test the nonlinear dependence of expected error on position in the zone by evaluating the expected error equation (5) for every realized position in the zone. After sorting realized positions from the lowest to the highest, we get the solid lines in Figure 4.\(^7\) X-axis on each graph is the position of inflation inside the zone, \( \tau_t = 2(\pi_{t-1} - C)/(U - L) \), 1 indicates the upper limit of target zone, -1 indicates the lower limit, and 0 indicates the center of the target zone. We reject the null hypothesis of the nonlinear dependence if the 90% highest posterior density interval (HPDI) around the estimates, the dashed lines in the Figure 4, contains zero for all observations.

\(^5\)If a true data generating process of inflation is a monthly price change, modeling dependent variable as a one year price change in a linear AR model leads to autocorrelation of errors. Autocorrelated errors result in efficiency loss and produce larger standard errors. Our residuals are close to the white noise.

\(^6\)The United Kingdom’s target series was changed in the middle of October 2003 from RPIX to harmonized consumer price index (HCPI).

\(^7\)We report the median, the mean is less informative because many estimates are exactly equal to zero.
Inside the target zone, the nonlinear dependence is not significant for all five countries. Between -1 and 1, the estimates of the expected error and the upper and lower bounds of the highest posterior density interval are zero. Outside the zone, below -1 or above 1, evidence of the nonlinear dependence is weak. Even though for all countries the estimates of the expected error are non-zero, they are barely significant. For Australia, Canada, and Sweden, the highest posterior density intervals contain zero; for the United Kingdom and New Zealand, the intervals are close to zero.

Significant anticipated interventions are an additional evidence of the nonlinear dependence. While the expected error line is constructed for all time periods, the measure of anticipated intervention strength is constructed for every time period giving more detailed evidence of the nonlinear dependence. Figure 5 reports our measure of anticipated intervention strength. The solid line reports the mean of anticipated intervention strength; the dashed lines report the 90% highest posterior density interval. An anticipated intervention significantly differs from zero when the restriction on probability to leave the target zone is binding $\tilde{\alpha}_t = \alpha^*$. In three countries – New Zealand, Sweden, and the UK – we find significant effect of anticipated interventions. In New Zealand, agents anticipated that interventions up to 1992 would decrease inflation on average by 1.5%. In Sweden, during the second half of 1996, the second half of 1998 and the beginning of 2003 agents anticipated that interventions would increase inflation on average by 1%. In the UK, agents anticipated that the 2000 intervention would decrease inflation on average by 0.4%, while the 2004 intervention would increase inflation on average by 0.5%. Agents in Canada and Australia did not anticipated significant interventions.

The estimate of the restricted probability of the target variable to leave the zone $\alpha^*$ also determines the degree of the nonlinear dependence of expected error on the position in the zone. When $\alpha^*$ is equal to 1, the dependence is linear. More generally, an increase of $\alpha^*$ lowers the degree of nonlinearity. However, this interpretation should be used with caution. The same value of $\alpha^*$ may lead to two opposite conclusions depending on the maximum of the unrestricted probability. For example, assume that the target variable never breaks the bounds and that $\alpha^*$ equals 5%. For a target zone with bounds $[-1, 1]$, the maximum of the unrestricted probability to move outside the target zone is approximately 33%. The unrestricted probability is much higher than the restricted probability $\alpha^*$. The large difference between probabilities implies binding interventions and a high degree of nonlinearity. However, for a target zone with bounds $[-2, 2]$, the maximum of the unrestricted probability is the same as the restricted probability 5%. Interventions are not binding, and the error term is linear. Thus, to test for the nonlinear dependence, we should compare the estimate of $\alpha^*$ to the maximum of the unrestricted probabilities, $\max(\tilde{\alpha}_t) - \alpha^*$. If the HPDI of $\max(\tilde{\alpha}_t) - \alpha^*$ contains zero, we reject the hypothesis of nonlinear dependence of expected error on the position in the zone.\footnote{Y-axis on each graph is the position of inflation inside the zone. To facilitate the exposition we translate the position in the zone back into inflation.}
Based on the estimate of the restricted probability $\alpha^*$, for all five countries the nonlinear dependence is not significant. Table 1 reports both $\alpha^*$ in fifth row with 90% highest posterior interval in brackets and the result of comparing the restricted probability to the maximum of the unrestricted probabilities in sixth row. For all countries the upper bound of the highest posterior density interval of $\alpha^*$ is close to 1. In addition, for Australia, Canada, and Sweden, the highest posterior interval of the difference between restricted and maximum of unrestricted probabilities contains zero rejecting nonlinearity. For New Zealand and the United Kingdom, the lower bound of highest posterior density interval is almost zero.

We have done the robustness check for the obtained results. All results do not depend on whether inflation has monthly or quarterly frequency, or whether we restrict inflation to be stationary process or relax the stationarity restriction. Results are also not sensitive to the choice of lag length in equation (1).

We also have checked whether the weak evidence in support of expectation anchoring can be attributed to the weakness of the proposed methodology. In appendix, we illustrate that the methodology is quite successful, when we apply it to testing the expectation anchoring for the exchange rate targeting policy in Norway. We find very strong evidence of expectations anchoring. First, the estimate of expected error as a function of position in the zone has a significant S-shape. Second, the estimate of agents anticipation of central bank interventions are significant and coincide with actual interventions. Finally, the estimate of the restricted probability $\alpha^*$ is essentially zero.

7 Concluding Remarks

We propose the expectations anchoring test for the inflation targeting policy that is based on the Krugman model. The test focuses on the fact that targeting modifies the link between realized and expected inflation for the very high and very low levels of inflation. The test avoids the critique of the benchmark approach, because it does not require comparison between targeting and non-targeting countries. In addition to the test, we can measure the degree of expectation anchoring and the strength of anticipated central bank interventions.

We find weak evidence of expectations anchoring in Australia, Canada, New Zealand, Sweden, and the United Kingdom. We attribute our finding to a credibility problem. Central banks often break their inflation targeting promises. For example, the central banks of Australia and Sweden allow large (as far as twice the size of the target zone) and long (as much as a year) breaks of the bounds. Such breaks undermine agents confidence in the inflation targeting promise: High inflation results in high expected inflation.
Appendix: Exchange Rate Expectations Anchoring in Norway

We illustrate the methodology with the daily Norwegian exchange rate index from October 1, 1986, to June 17, 1988, Figure 3. During the analyzed period, the exchange rate index was allowed to fluctuate within ±2.25% from its central parity. According to Mundaca (2001), Norges Bank (Central Bank of Norway) intervened at the boundaries of target zone between October 1, 1986, and June 17, 1988. The analysis of the daily Norwegian exchange rate is a good testing ground for checking the ability of the proposed methodology to test expectations anchoring. To the best of our knowledge, in this sample, all previous studies find strong evidence of nonlinear dependence of the expected exchange rate on the position in the zone.

Let \( e_t \) denote the process for Norwegian exchange rate index. Then normalized target is \( \tau_t = 2(e_t - C_t)/(U_t - L_t) \). Following empirical exchange rate literature, we assume that exchange rate has a unit root and estimate

\[
\Delta \tau_t = c + \epsilon_t,
\]

We find strong evidence of expectations anchoring. All three alternative tests confirm this result. First, the estimate of expected error as a function of position in the zone is significantly negative close to the upper bound, a lower right graph in Figure 4, while it is significantly positive close to the lower bound. Second, there are four periods when agents anticipated significant interventions in Figure 5. Agents anticipated that interventions between December 1986 and January 1987 and between December and January 1988 would decrease the exchange rate, while interventions between July and October 1987 and between May and June 1988 would increase the exchange rate. The size of anticipated intervention is on average 0.6%, or 10% of the target zone. The periods of anticipated interventions are very close to the periods of actual interventions reported by Mundaca (2001). Finally, the estimate of \( \alpha^\ast \) is almost zero, and the highest posterior interval of the difference between restricted and maximum of unrestricted probabilities is far above zero, with the lower bound 0.4.

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10For most recent examples, see Lundberg and Terasvirta (2006) or Korenok and Radchenko (2005).
11See for example Lundberg and Terasvirta (2005), Bekaert and Gray (1998).
References


### Table 1: Parameter Estimates

<table>
<thead>
<tr>
<th>Prior</th>
<th>Canada</th>
<th>Sweden</th>
<th>United Kingdom</th>
<th>Australia</th>
<th>New Zealand</th>
<th>Norway</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0) N(0, 1)</td>
<td>-0.033</td>
<td>-0.068</td>
<td>-0.093</td>
<td>0.046</td>
<td>0.205</td>
<td>-0.010</td>
</tr>
<tr>
<td>(a_1) N(0.98, 1)</td>
<td>0.891</td>
<td>0.940</td>
<td>0.943</td>
<td>0.815</td>
<td>0.564</td>
<td>0.997</td>
</tr>
<tr>
<td>(a_2) N(0, 1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.114</td>
<td>0.114</td>
<td></td>
</tr>
<tr>
<td>(\sigma) IG(0.005, 1)</td>
<td>0.228</td>
<td>0.341</td>
<td>0.257</td>
<td>0.754</td>
<td>0.390</td>
<td>0.092</td>
</tr>
<tr>
<td>(\alpha^*) Beta((\gamma, \delta))</td>
<td>0.806</td>
<td>0.955</td>
<td>0.748</td>
<td>0.768</td>
<td>0.767</td>
<td>0.032</td>
</tr>
<tr>
<td>(\max(\alpha_1) - \alpha^*)</td>
<td>[0.649, 0.979]</td>
<td>[0.915, 0.999]</td>
<td>[0.571, 0.940]</td>
<td>[0.535, 0.977]</td>
<td>[0.589, 0.963]</td>
<td>[0.007, 0.57]</td>
</tr>
</tbody>
</table>

**Notes:** Standard errors are reported in parenthesis, 90% highest posterior density intervals in square brackets.
Notes: Conditional expectations of the error term as function of the position in the target zone are constructed for fixed $\sigma = 0.2$, $a(L) = 0$ and $\alpha^* = 0.2$. On X-axis 1 indicates the upper limit of target zone, -1 indicates the lower limit, and 0 indicates the center of the target zone.
Figure 2: Probability of the Target to Break the Target Zone

Notes: $\tilde{\alpha}_t$ is a probability of breaking the target if agents anticipate no central bank interventions, $\alpha^*$ is a constant level at which agents anticipate monetary policy to intervene, $\alpha_t$ is a probability of breaking the zone, which accounts for anticipated monetary policy interventions. All probabilities are constructed for fixed $\sigma = 0.2$, $a(L) = 0$ and $\alpha^* = 0.2$. On X-axis 1 indicates the upper limit of target zone, -1 indicates the lower limit, and 0 indicates the center of the target zone.
Figure 3: Dynamics of the Target in the Target Zone: Historical Observations

Notes: Y-axis on each graph is the position of inflation (exchange rate for Norway) inside the zone, $\tau_t = 2(\pi_{t-1} - C)/(U - L)$, 1 indicates the upper limit of target zone, -1 indicates the lower limit, and 0 indicates the center of the target zone.
Figure 4: Estimates of Conditional Expectation of the Error Term

Notes: The estimated relation between the expected error term and the position of inflation (exchange rate for Norway) inside the target zone. Solid lines denote the median of estimates (mean is less informative because a large number of estimated error terms in MCMC draws are zero), dashed lines denote the upper and lower bound of 90% highest posterior density interval. See also notes for Figure 1.
Figure 5: Measure of Anticipated Intervention Strength

Notes: Inverted distance between probability of breaking the target zone without interventions and probability of breaking the target zone with interventions. For further details see section 3.3. Solid lines denote the median estimate of intervention strength, dashed lines denote the upper and lower bound of 90% highest posterior density interval. See also notes for Figure 3.