Economics 661

Public Economics

Professor Lauura Razzolini

Spring 2006

Lecture 2

Outline

I. Utility and profit maximization
II. Case for competitive markets
III. Costs of intervention

Micro theory review outline

- Review of micro theory (Gruber Ch. 2)
  - Individual utility maximization (➔ D curve)
    - Preferences and the MRS
    - Budget constraint (constrained maximization)
    - Equilibrium
  - Firm's profit maximization (➔ S curve)
  - Efficiency vs. welfare
    - 1st and 2nd Welfare Theorems
    - Consumer, producer, and social surplus

Utility

- An individual chooses among consumption goods *x and y*
  - You can generalize to more than two goods!

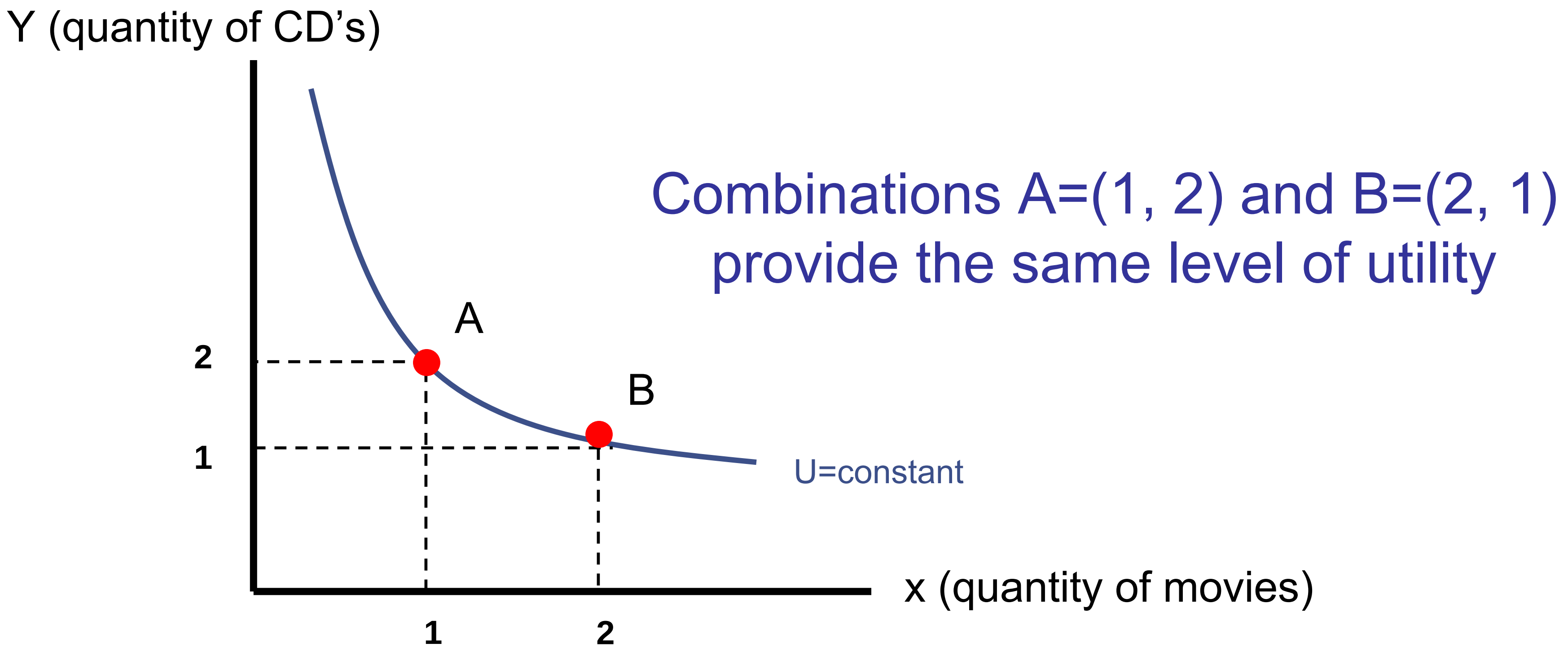
- The individual's rankings can be shown by a utility function of the form:

$$\text{utility} = U(x, y)$$

where u(.) is some mathematical function

Indifference curves

- Sets of consumption bundles among which the individual is indifferent

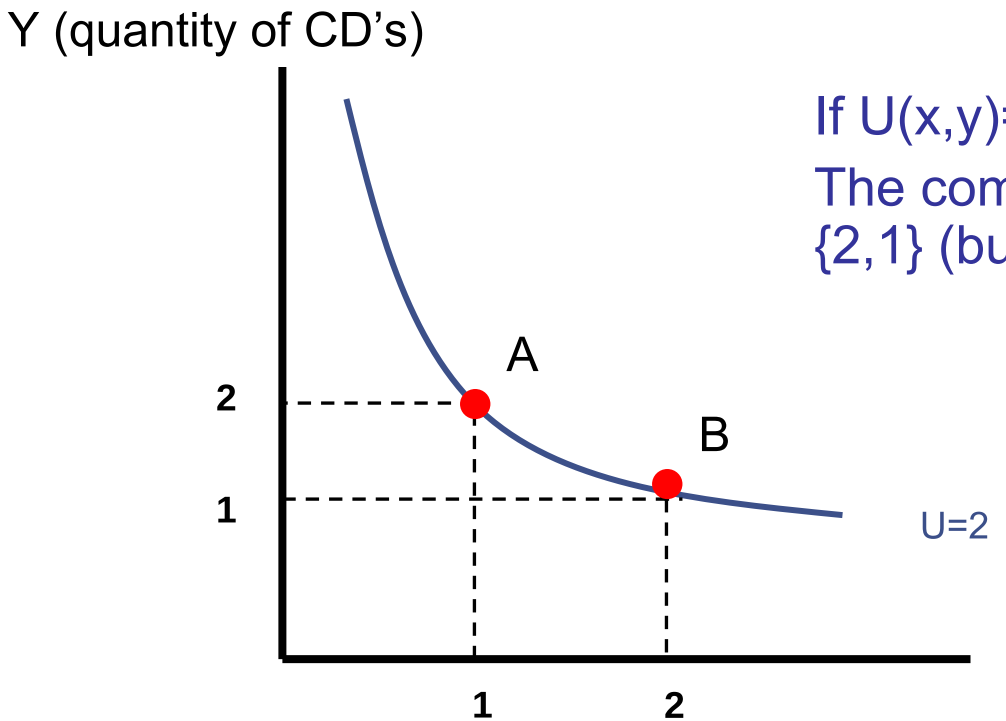


Indifference curves (ICs)

- Sets of consumption bundles among which the individual is indifferent
  - Indifference curves are always downward sloping (otherwise violation of the more-is-better assumption)

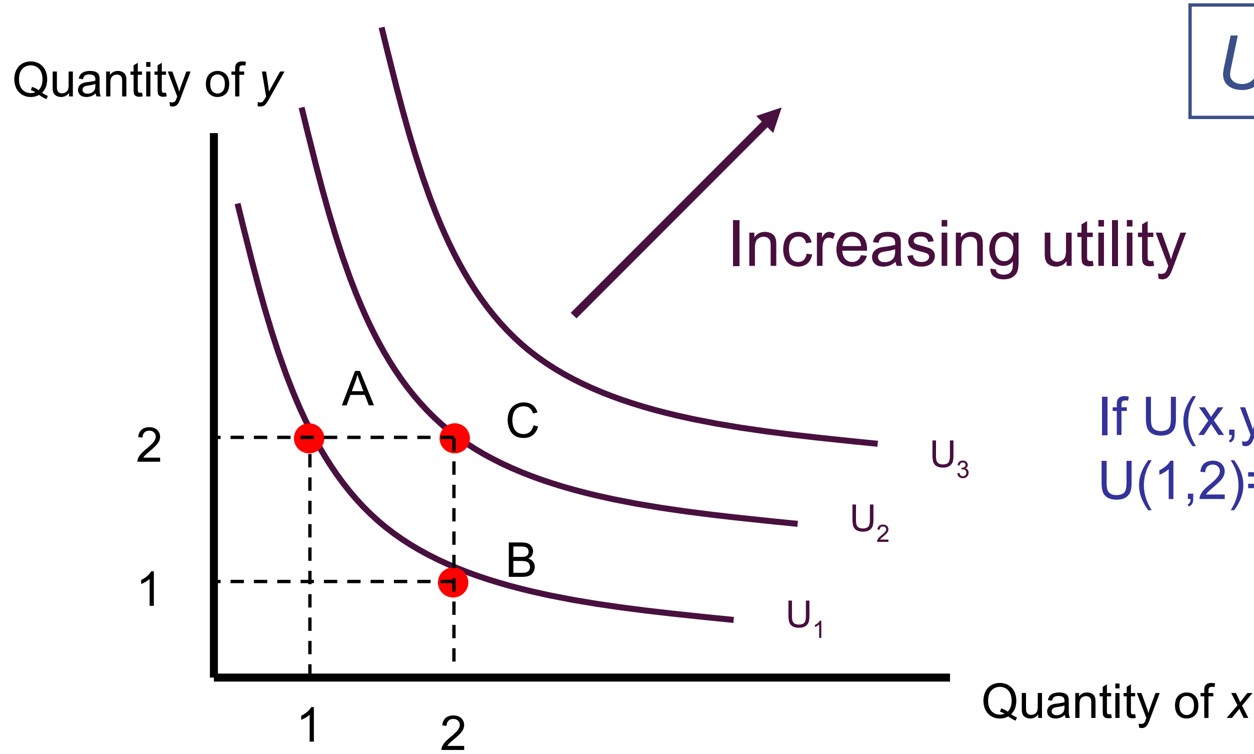
If U(x,y)=xy, then U(1,2)=U(2,1)=2

The combinations {1, 2} (bundle *A*) and {2,1} (bundle *B*) both give 2 “utils.”



Indifference curve map

- Individuals prefer higher indifference curves because they represent bundles that have more of both goods



$U_1 < U_2 < U_3$

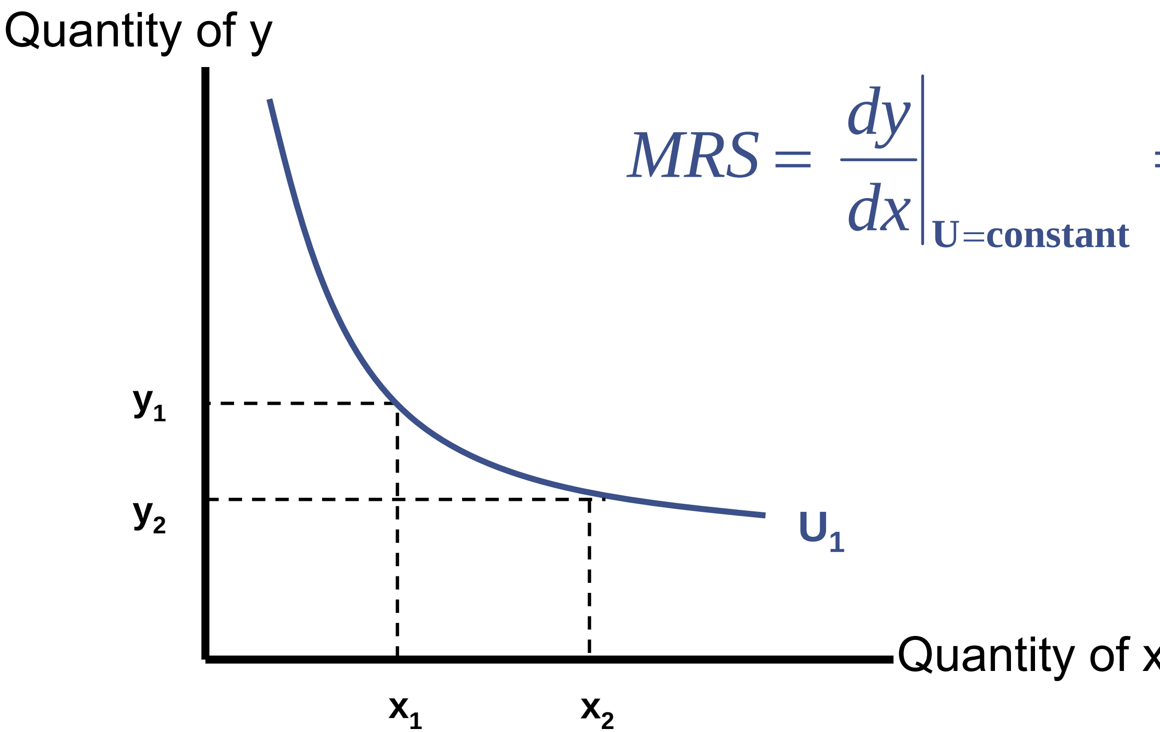
If U(x,y)=xy, then
U(1,2)=U(2,1)=2 < U(2,2)=4

How are indifference curves derived?

- Set utility equal to a constant level and figure out the bundles of goods that get that utility level.
- For $U = xy$, how would we find the bundles for the indifference curve associated with 25 utils?
  - Set $25 = xy$,
  - Yields $y = 25/x$,
  - Or bundles like {1,25}, {1.25,20}, {5,5}, etc.

Marginal rate of substitution

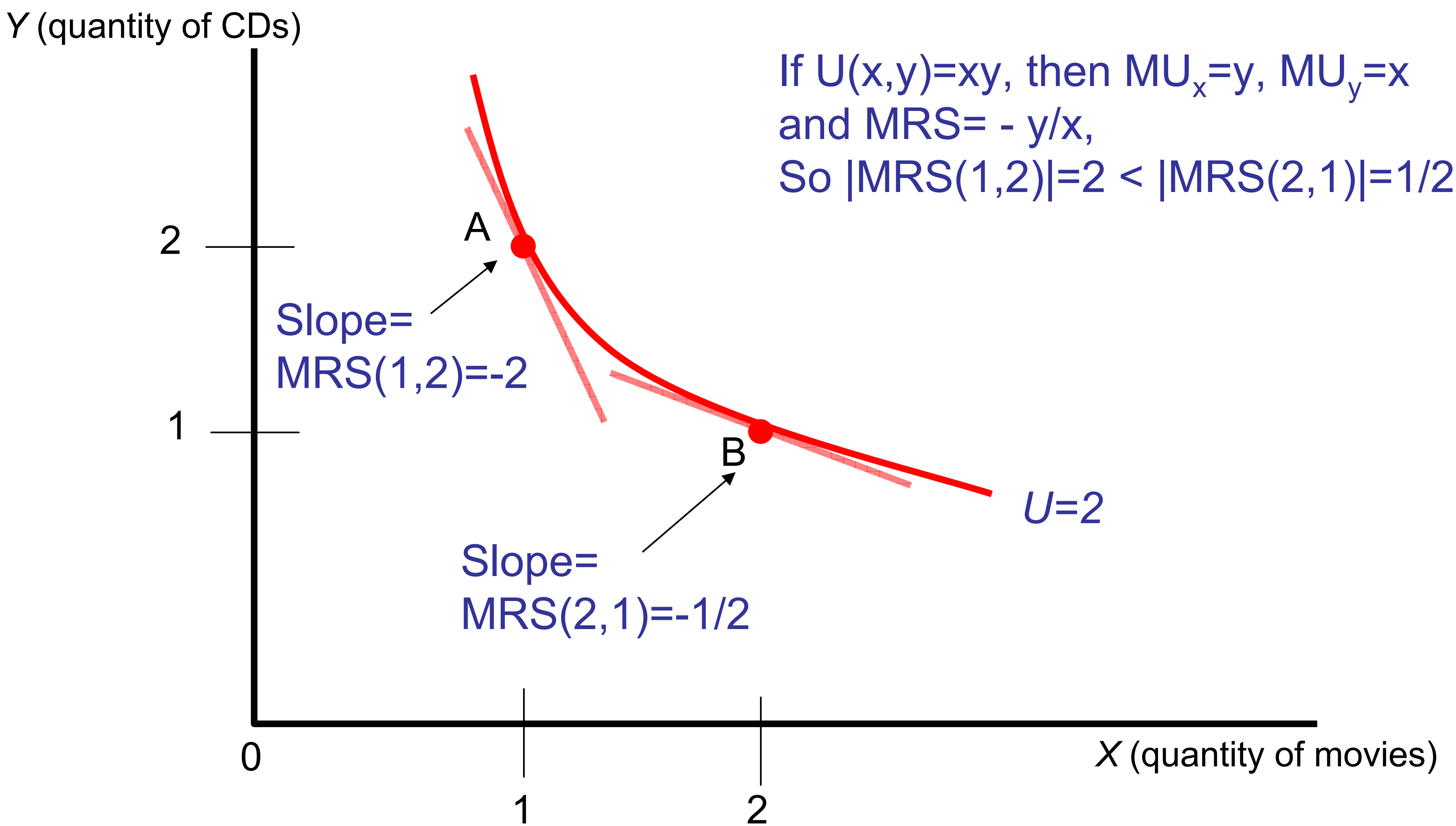
- The slope of the indifference curve is called the *MRS*, and is the rate at which the consumer is willing to trade off good *y* for good *x*.
- MRS is the negative of the ratio of marginal utilities



$$MRS = \left.\frac{dy}{dx}\right|_{\mathbf{U=constant}} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{MU_x}{MU_y}$$

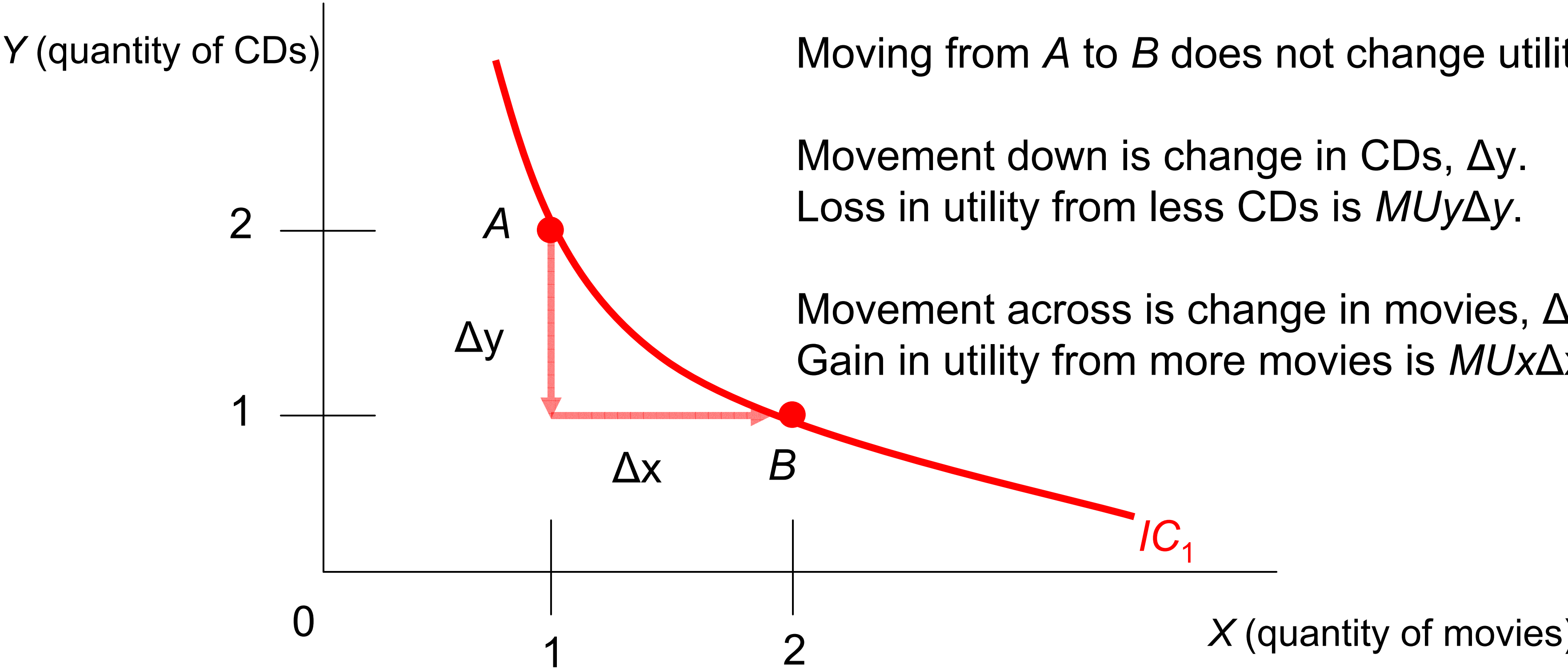
***MU*$_x$** is the additional increment to utility from consuming an additional unit of good x

- Positive
- Diminishing in x



- ***MRS*** is diminishing (in absolute terms) as we move along an indifference curve
    - Remember that MU_x drops and MU_y increases along an IC (more x, less y)

- This means that the consumer is willing to give up fewer CD's to get more movies when she has more movies (*B*) than when she has less movies (*A*).



Moving from *A* to *B* does not change utility

Movement down is change in CDs, Δy.
Loss in utility from less CDs is *MUyΔy*.

Movement across is change in movies, Δ*x*.
Gain in utility from more movies is *MUx*Δx.

Must have *MUyΔy=MUxΔx* because we are on the same indifference curve. Simply rearrange the equation to get the relationship between *MRS* and marginal utilities

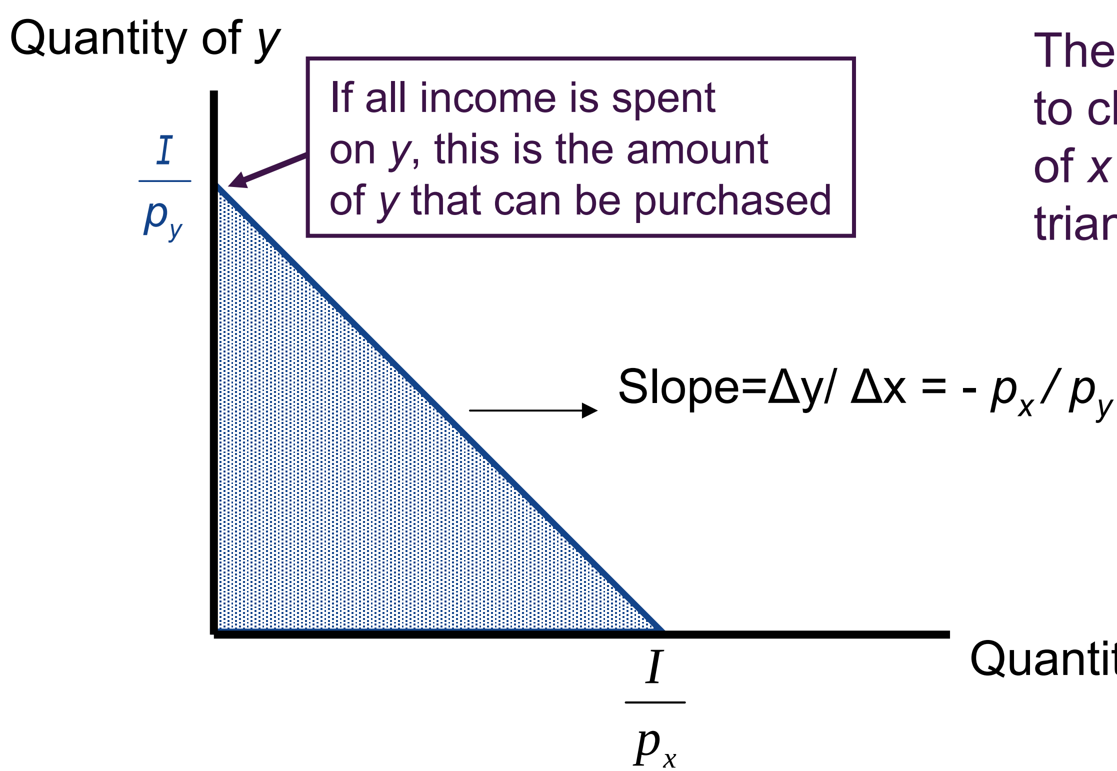
Mathematically, set the total differential of U(x,y) equal to zero:

$$dx\frac{\partial U}{\partial x} + dy\frac{\partial U}{\partial y} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = MRS$$

The budget constraint

- Assume that an individual has *I* dollars to allocate between good *x* and good y

$$p_x x + p_y y \leq I$$

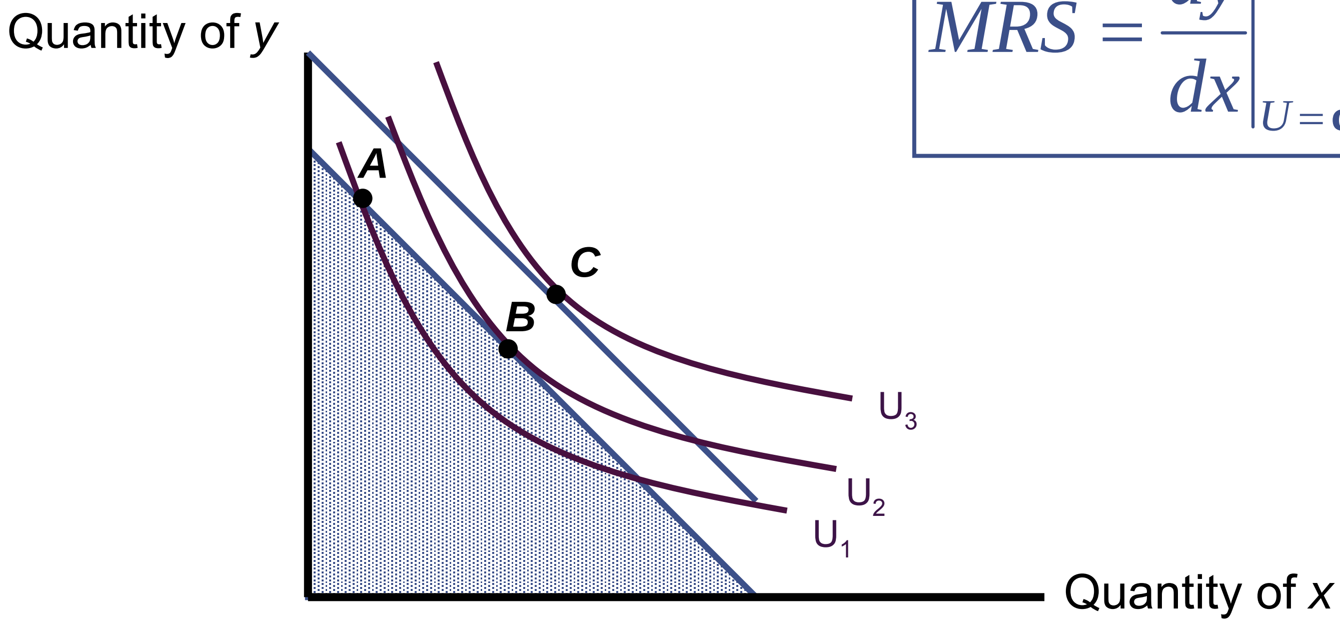


The individual can afford to choose only combinations of *x* and *y* in the shaded triangle

Example: *I*=100, p_x=5, p_y=10
Slope: - 5 / 10 = -1/2
Horizont. intercept: 100/5=20
Vertical intercept: 100/10= 10

Utility maximization

- Add the individual's utility map to show the utility-maximization process
- the utility maximizing choice occurs where the indifference curve is *tangent* to the budget constraint.: the slope of the indifference curve equals the slope of the budget constraint



$$MRS = \left.\frac{dy}{dx}\right|_{U=\text{constant}} = -\frac{p_x}{p_y}$$

Ex.: Calculus treatment of constrained utility maximization problem

U(x, y) = xy

$p_x = 5$, $p_y = 10$, I=100

Problem: $\max_{x,y} U(x,y) = xy$

s.t. *5x* + 10*y* = 100

Step 1: substitute BC into utility function

y = 10 – 1/2x

$\max_x x(10-1/2x)=10x-1/2x^2$

Step 2: take the derivative w.r.t. *x* and set equal to 0

10-2/2x=0 → x=10

Step 3: plug *x* into BC to find optimal *y*

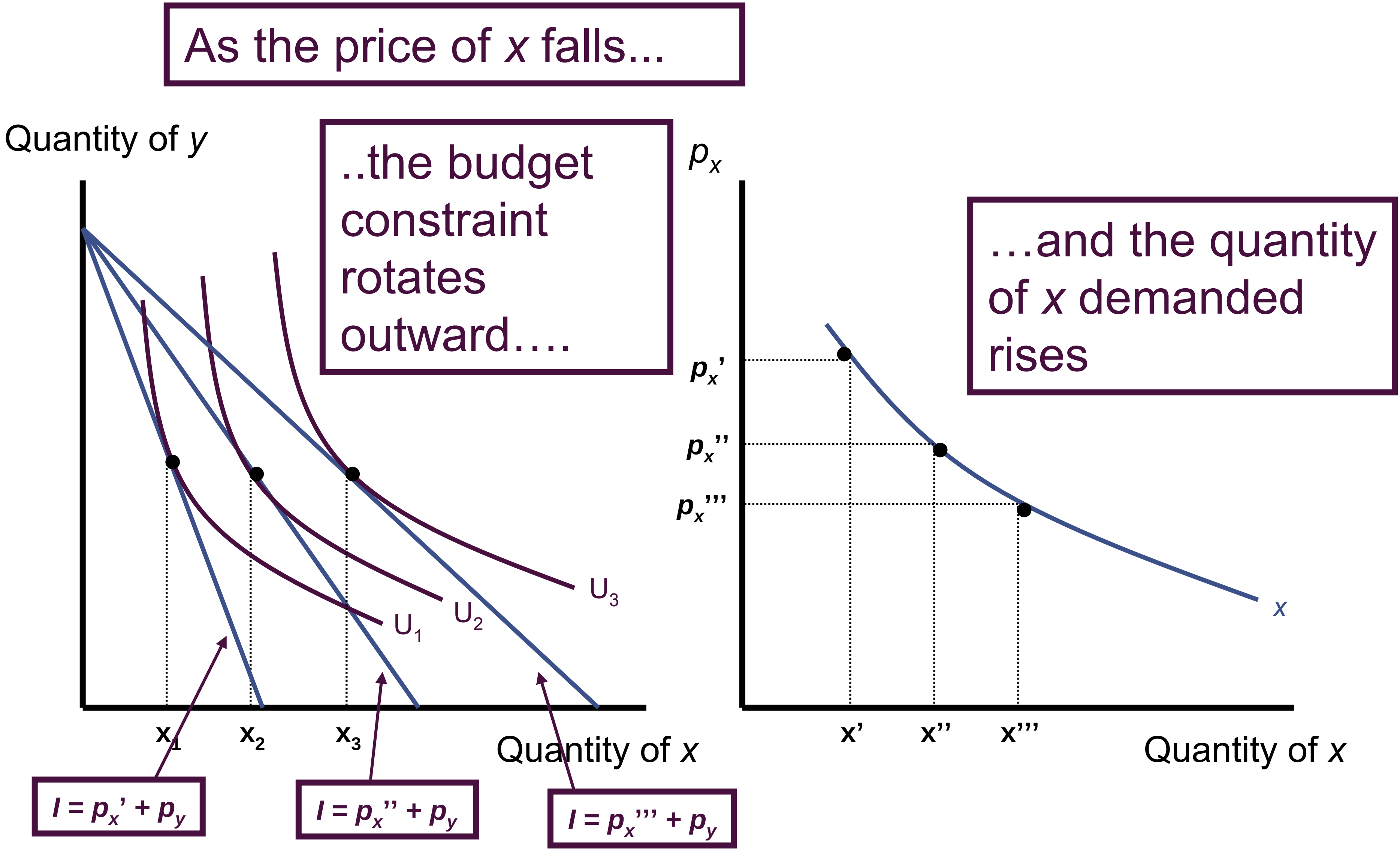
y = 10 – 1/2x → y=10-1/2*10 → *y=5*

(x*,y*)=(10,5)

Next: find MRS (at consumer's optimum) and check your results!

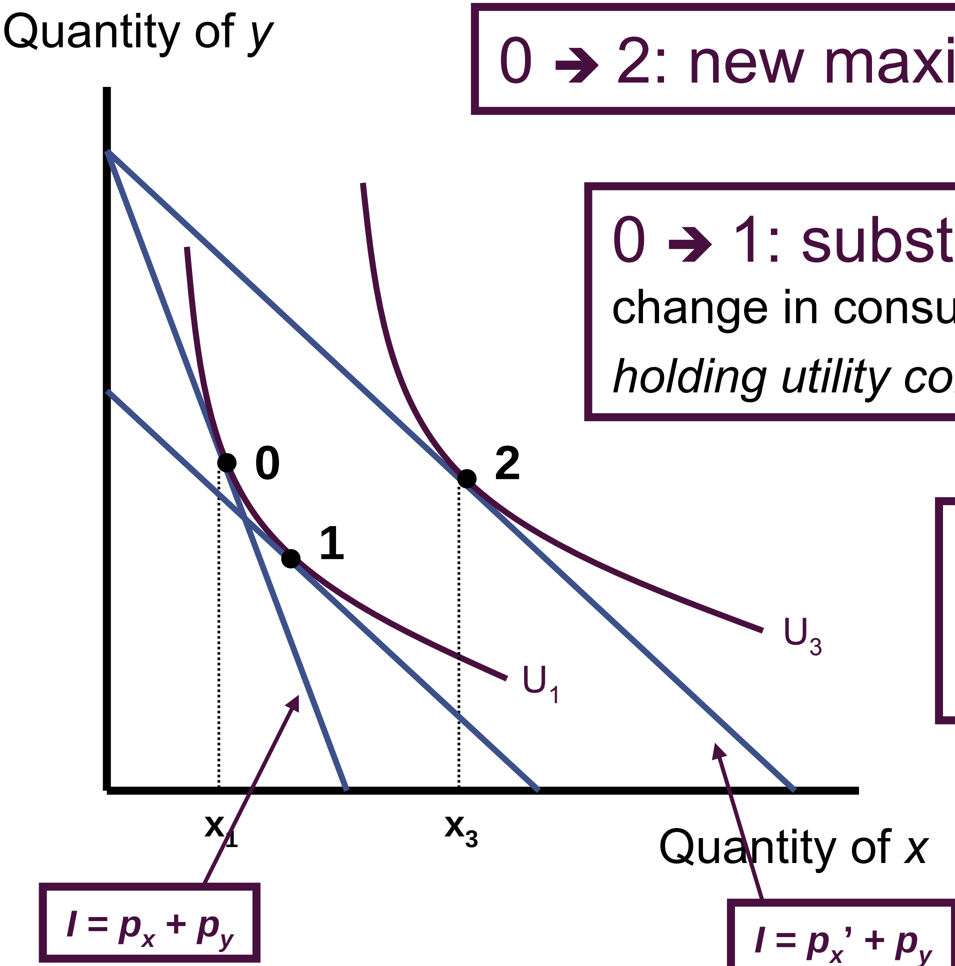
$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{y}{x} \quad so \;\; MRS(10,5) = -\frac{1}{2} = -\frac{p_x}{p_y}!$$

The individual's demand curve



Income and substitution effects

The price of x falls from p_x to p_x'



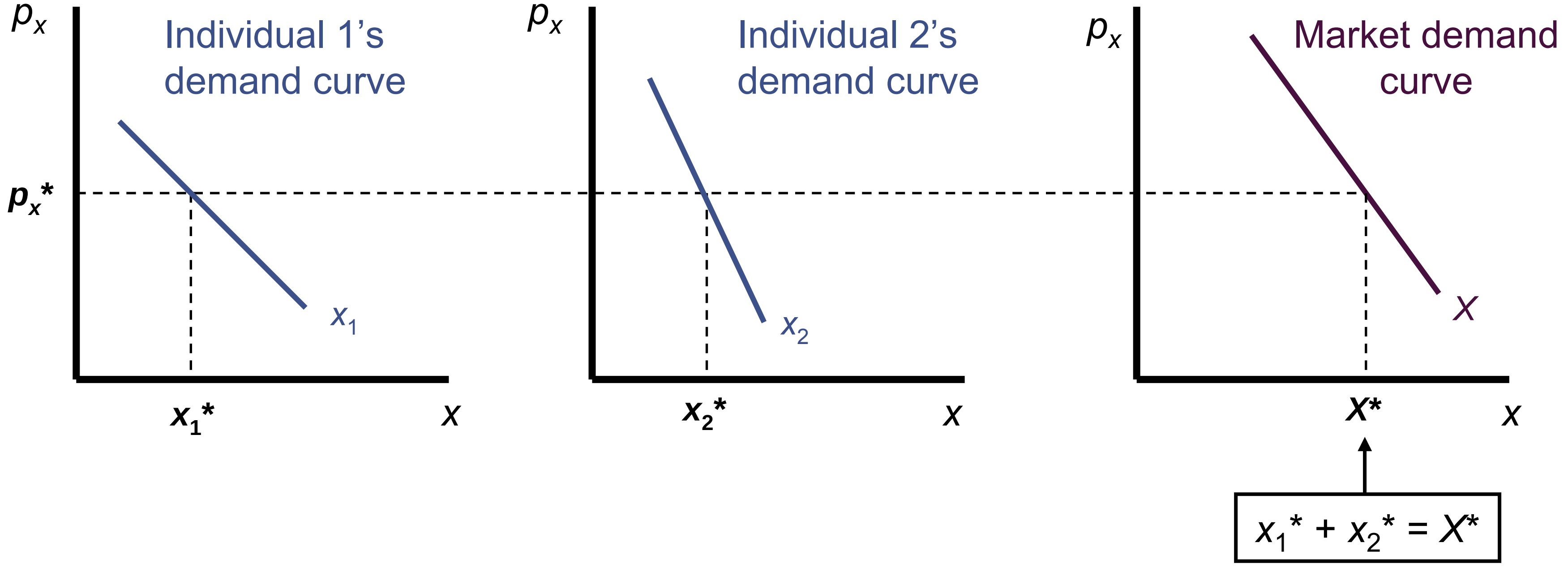
0 ➔ 2: new maximization bundle

0 ➔ 1: substitution effect:
change in consumption due to change in relative prices, *holding utility constant*.

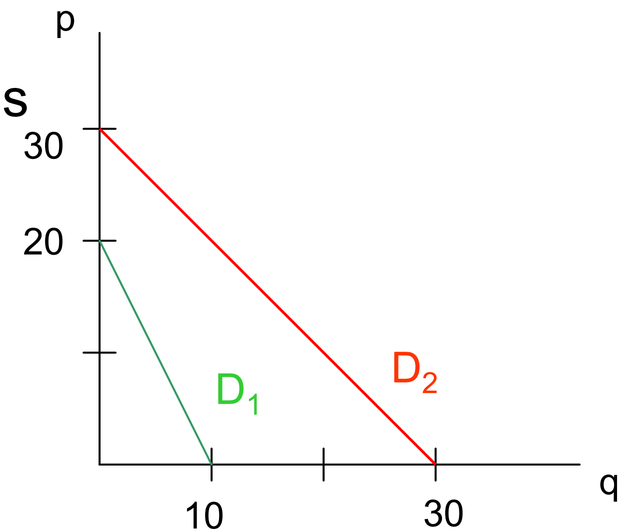
1 ➔ 2: income effect:
change in consumption due to feeling "richer" after price drop.

Market demand

- To derive the market demand curve, we sum the quantities demanded at every price
- Horizontally sum the individual demand curves



Ex.: Calculus of aggregate demand



2 consumers with demand curves/ MB schedules

$MB_1=$	$20-2q_1$	for	$0 \le q_1 \le 10$
	0	for	$q_1 > 10$
$MB_2=$	$30-q_2$	for	$0 \le q_2 \le 30$
	0	for	$q_2 > 30$

Total demand: in case of a rivalrous good, $Q=q_1+q_2$

Step 1: Rewrite demand schedules as quantity as function of price

$q_1=$	$10-1/2p_1$	for	$0 \le p_1 \le 20$
	0	for	$p_1 > 20$
$q_2=$	$30-p_2$	for	$0 \le p_2 \le 30$
	0	for	$p_2 > 30$

Ex.: Calculus of aggregate demand

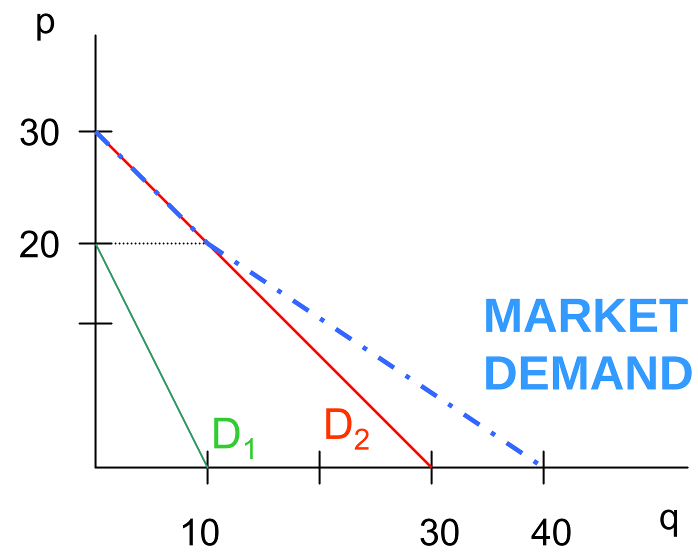
Step 2: sum the individual demand curves (check the ranges of prices!)

Total demand curve:

Q=	40-3/2p	for	0≤p≤20
	30-p	for	20≤p≤30
	0	for	p>30

Total inverse demand → MSB schedule:

p=	80/3-2/3Q	for	10≤Q≤40
	30-Q	for	0≤Q≤10
	0	for	Q>30



For a rivalrous good, MSB=MB_i or MB_j

Given a marginal unit of the good only one consumer gets it. So the MSB is the MB of the marginal consumer.

Mathematically

- The individual's objective is to maximize

$$U(x,y)$$

subject to the budget constraint

$$I \leq p_x x + p_y y$$

- Substitute for *y* using the budget constraint:

$$Max_x U\left(x, \frac{I - p_x x}{p_y}\right)$$

- First-order condition for an interior maximum:

$$\frac{\partial U}{\partial x} - \frac{\partial U}{\partial y}\frac{p_x}{p_y} = 0$$

$$|MRS| = \frac{\partial U / \partial x}{\partial U / \partial y} = \frac{p_x}{p_y}$$

Demand functions

- Individual *i* 's demand for *x* and *y* will depend on all prices and income

$$x_i^* = x_i(p_x, p_y, I)$$

$$y_i^* = y_i(p_x, p_y, I)$$

- The market demand for *X* is the sum of individuals' demands

$$X = \sum_{i=1}^{N} x_i(p_x, p_y, I_i)$$

The firm's profit maximization problem: perfect competition

Profits = revenues – costs

Benefit from producing last unit = MR = *p*

Cost of producing last unit = MC

What is firm's supply curve?

MC (when AC exceeds MR)

What condition is satisfied at q^*?

MR = MC

Mathematically

- Economic profits for a price-taking firm:

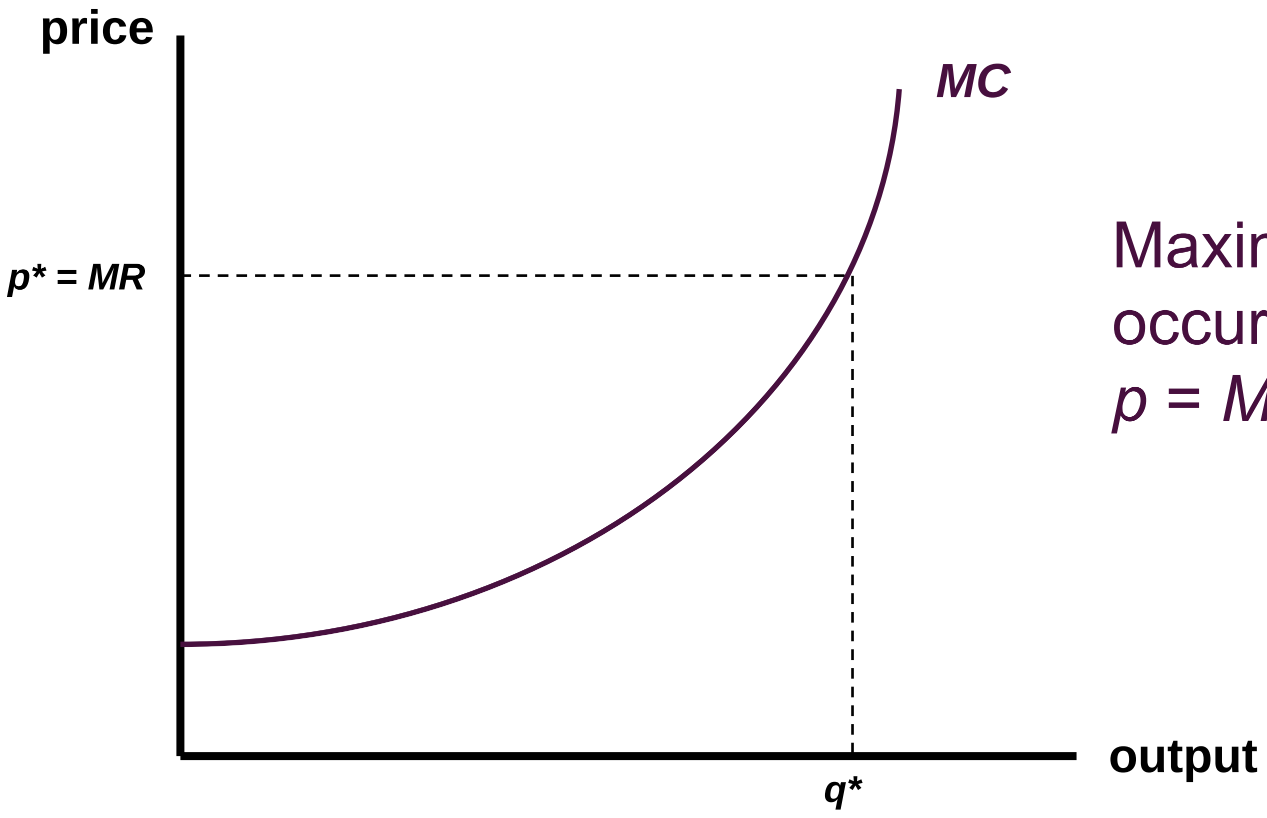
$$\pi(q) = R(q) - C(q) = p \cdot q - C(q)$$

- Necessary condition for profit maximization:

$$\frac{d\pi}{dq} = p - \frac{dC}{dq} = 0$$

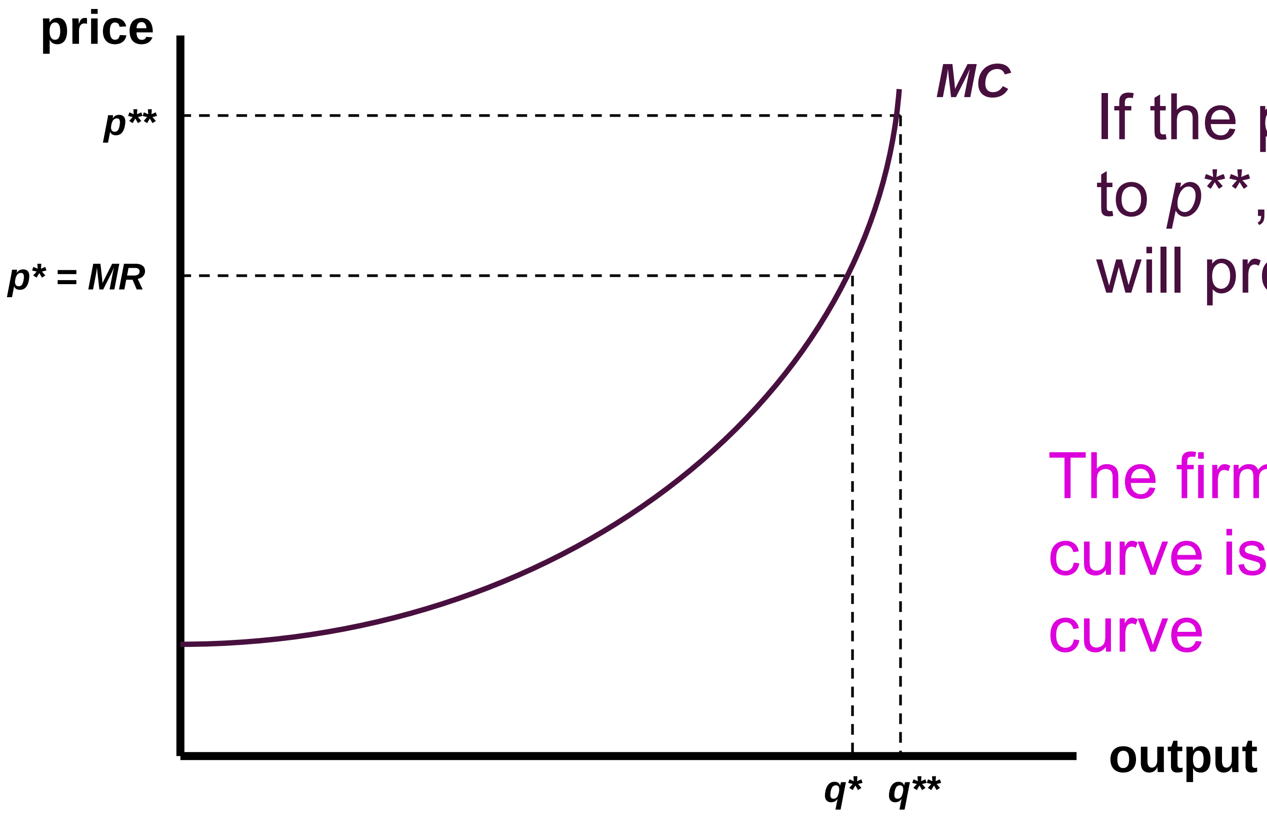
$$p = MC(q)$$

Supply by a price-taking firm



Maximum profit occurs where $p = MC$

Supply by a price-taking firm



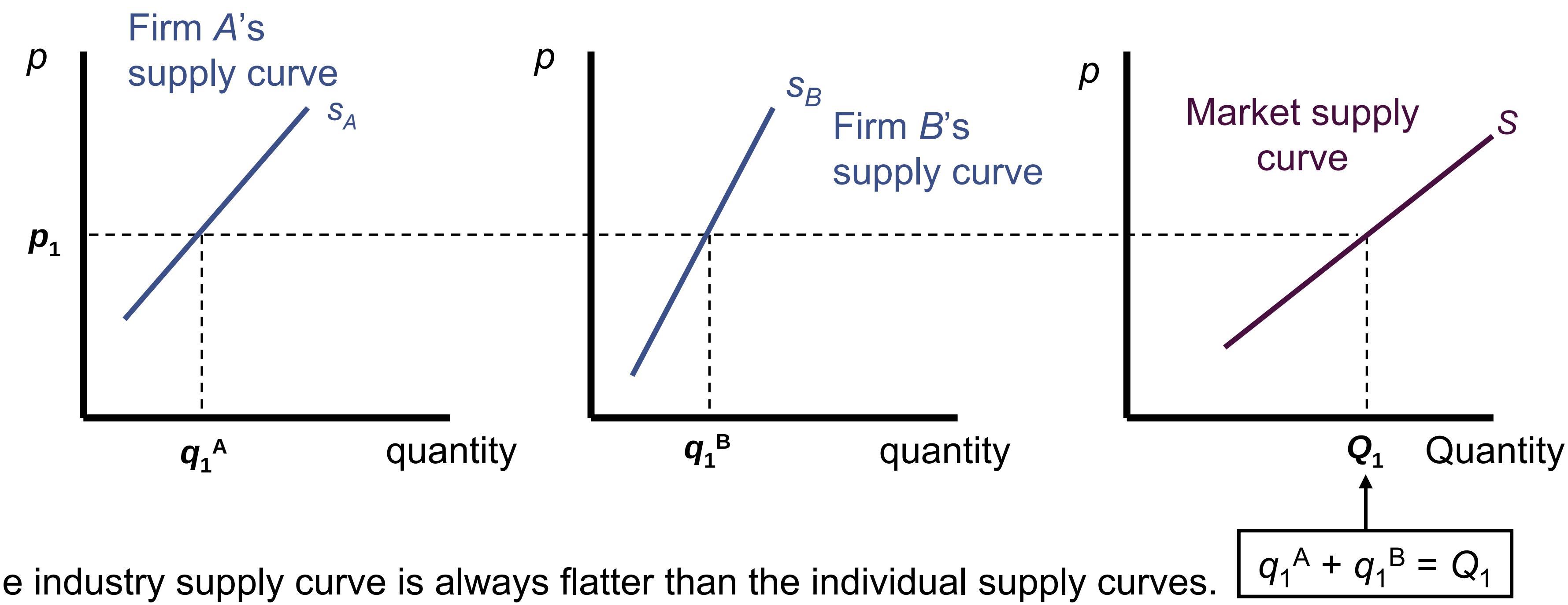
If the price rises to *p***, the firm will produce *q***

The firm's supply curve is the *MC* curve

Market supply curve

- To derive the market supply curve, we horizontally sum individual supply curves

$$Q_s(p) = \sum_{i=1}^{n} q_i(p)$$



The industry supply curve is always flatter than the individual supply curves.

For every p, the total quantity supplied by the industry is higher than each individual's production

Ex.: Calculus of aggregate supply

10 firms, each with

$MC_i=5+2q_i$

Each firm produces up to the point where $MC_i=p$

Step 1: Rewrite supply schedules as quantity as function of price

Inverse firm supply function

$q_i=$ 1/2p-5/2 for $p\geq5$

0 for $p<5$

Step 2: sum the individual supply curves

Total supply curve:

$Q = 10*q_i =$ 5p-25 for $p\geq5$

0 for $p<5$

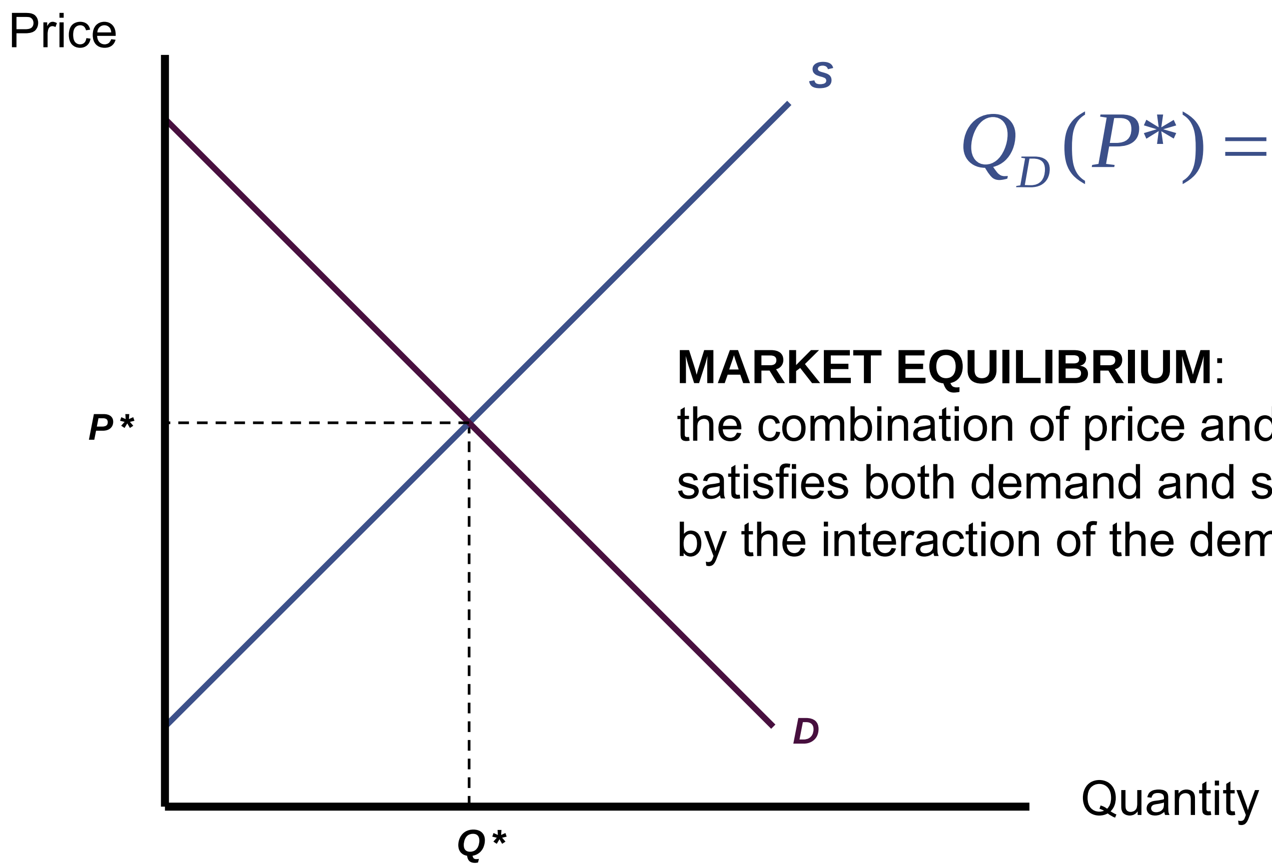
Total inverse supply → MSC schedule:

p= 1/5Q-5 for $Q\geq0$

The case for markets

- The competitive market equilibrium maximizes social surplus
- Social surplus is the total additional value obtained by market participants by being able to make market transactions

Competitive market equilibrium



$$Q_D(P^*) = Q_S(P^*)$$

MARKET EQUILIBRIUM:
the combination of price and quantity that satisfies both demand and supply, determined by the interaction of the demand and supply curves

Ex.: Calculus of market equilibrium

Total inverse demand → MSB schedule:

p= 20-Q

Total inverse supply → MSC schedule:

p= 10+Q

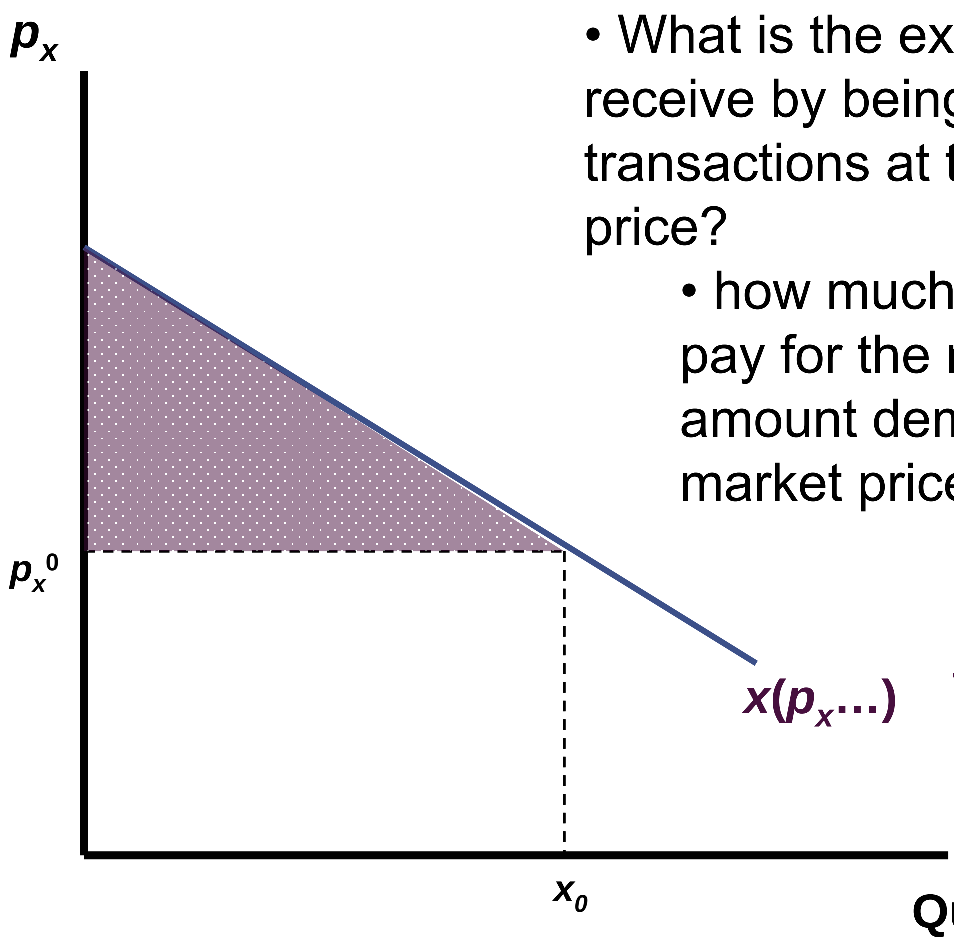
EQUILIBRIUM:

20-Q= 10+Q → 2Q=10 → Q*=5

P*=20-Q*=10+Q*=15

→ (Q*,p*)=(5,15)

Consumer surplus



- What is the extra benefit consumers receive by being able to make market transactions at the prevailing market price?
  - how much would they be willing to pay for the right to consume the amount demanded at the prevailing market price?

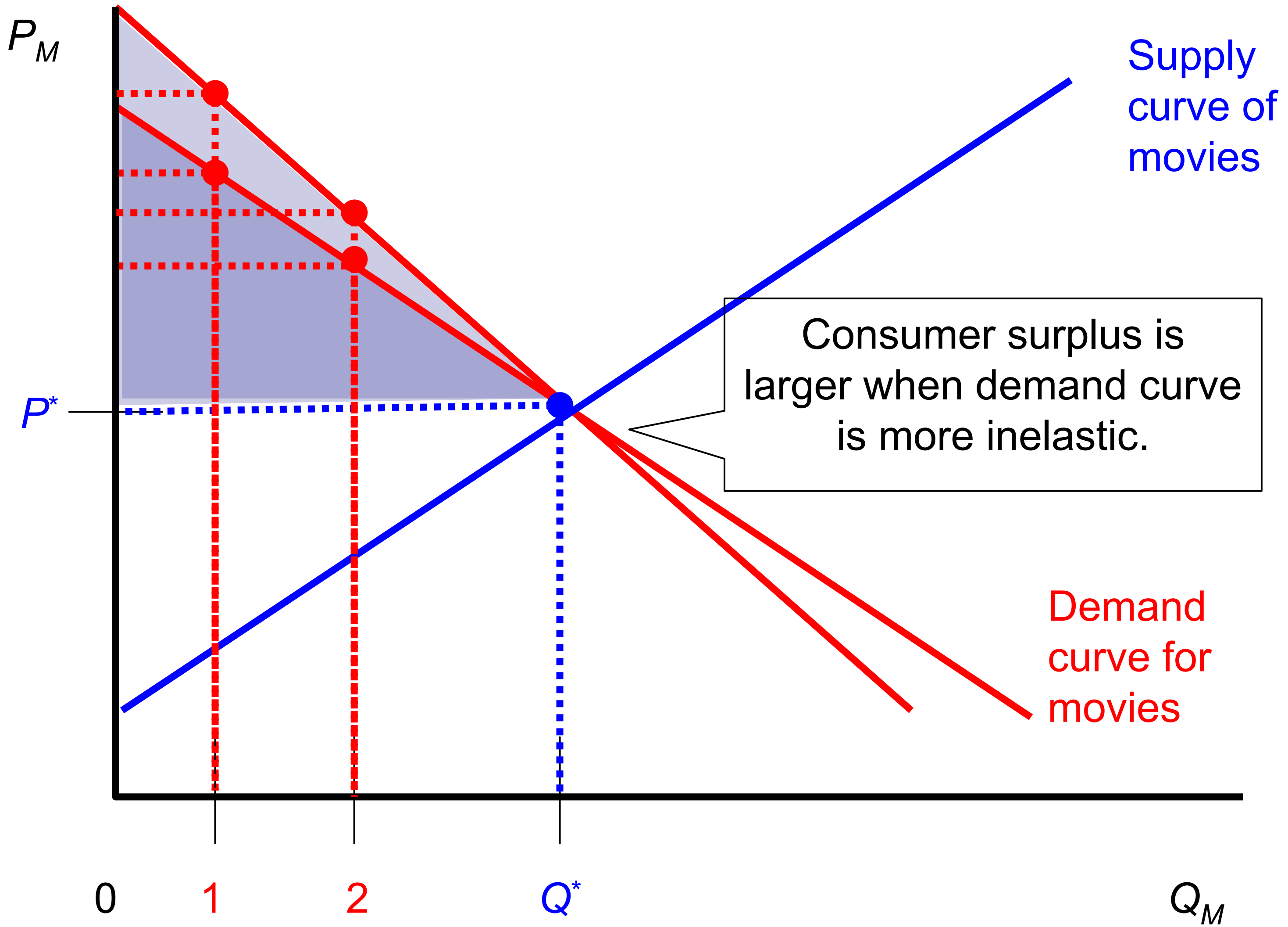
Consumer's surplus is the area above price and below demand

Elasticity of demand

- A key feature of demand analysis is the ***elasticity of demand***. It is defined as:

$$\varepsilon_D = \frac{\Delta Q_D / Q_D}{\Delta P / P}$$

- That is, the percent change in quantity demanded divided by the percent change in price.
- A vertical demand curve is ***perfectly inelastic***
  - Elasticity of demand is zero—quantity does not change as price goes up or down.
- A horizontal demand curve is ***perfectly elastic***
  - Elasticity of demand is negative infinity—quantity changes infinitely for even a small change in price.
- Consumer surplus is determined by market price and the elasticity of demand:
  - With inelastic demand, demand curve is more vertical, so surplus is higher.
  - With elastic demand, surplus is lower.



P_M
Supply curve of movies
Consumer surplus is larger when demand curve is more inelastic.
P^*
Demand curve for movies
0
1
2
Q^*
Q_M