After our paper was published, we received correspondence from several authors who published some of the results earlier. We regret not referencing the articles listed below and warmly recommend them as a supplement to ours. As it happens, the question of differentiability of modifications of Thomae's functions has been considered quite a few times in past MONTHLY articles. In particular, results very similar to our Propositions 3.1 and 4.1 appear in the literature.

In 1951, M.K. Fort [1] proved that a function on $\mathbb{R}$ with a dense set of discontinuities can only be differentiable on a set of first category. This implies that for every Thomae-type function there will be many irrationals on which it is not differentiable. The fact that Thomae's function is not differentiable anywhere is originally due to G.J. Porter [5]. The initial investigations into the differentiability of powers of the Thomae function are due to G.A. Heuer and an undergraduate group [2]. In this article, they use diophantine approximation theory to prove a result related to our Proposition 4.2. They also prove that any modification of Thomae's function must fail a Lipschitz condition on an uncountable dense set; this implies our Corollary 3.2. A few years later, J. E. Nymann [4] also considered powers of the Thomae function and obtained results comparable to those in [2]. Most recently, A. Norton [3] considered the modification of Thomae's function sending m/n to $1/2^n$ and proved a theorem concerning the size of the sets on which this function is infinitely differentiable or exactly k-times differentiable for every $k \in \mathbb{N}$.

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