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Book Review

An Introduction to Dynamical Systems: Continuous and Discrete, *R. Clark Robinson* Pearson Prentice Hall, Upper Saddle River, NJ, 2004, 672 pages, US$ 97.20, ISBN 0-13-143140-4.

Undergraduate texts on nonlinear dynamics traditionally place emphasis on differential equations with little or no mention of discrete dynamical systems. Although the use of difference equations as mathematical models has become increasingly popular in recent years, exposure to discrete systems is still neglected in many undergraduate mathematics curricula.

In the book, "An Introduction to Dynamical Systems: Continuous and Discrete" by R. Clark Robinson, differential and difference equations share equal time, with Chapters 1–7 focusing on continuous dynamical systems and Chapters 8–14 focusing on discrete dynamical systems. The organization of the text allows the instructor to emphasize theory or application. Each chapter consists of three parts: (i) opening sections in which main concepts are introduced, (ii) a section on applications, most of which are drawn from biology, chemistry, physics and economics and (iii) a section entitled Theory and Proofs. A typical chapter concludes with two to three dozen exercises, most of which are accessible to undergraduates who have some knowledge of linear algebra and differential equations.

The first seven chapters concern systems of ordinary differential equations, covering solutions of linear systems, nonlinear systems, existence and uniqueness, stability of equilibria, linearization about hyperbolic equilibrium points, Lyapunov functions, periodic orbits, bifurcations and chaos. Chapter 8 sets the stage for the study of iterated mappings, drawing motivating examples from applications to economics and population biology. Analysis of one-dimensional mappings is the focus of Chapter 9, which introduces fixed points, periodic orbits, stability and bifurcation theorems. Chapter 10 opens by introducing transition graphs and the Sharkovskii Theorem, followed by a discussion of how symbolic dynamics can be used to find orbits that exhibit complicated behavior. Chapter 11 provides a more in-depth study of invariant sets and chaotic attractors. Lyapunov exponents are introduced to quantify sensitive dependence on initial data and frequency measures are presented as a means of describing the distribution of iterates in invariant sets. Higher-dimensional maps are introduced in Chapter 12, which includes criteria for stability of fixed points and periodic orbits as well as statements of the stable manifold and Hartman–Grobman theorems. In Chapter 13, the Smale horseshoe map is used to motivate the study of invariant sets for higher-dimensional maps. Higher-dimensional analogues of concepts presented in earlier chapters (symbolic dynamics, chaotic attractors and Lyapunov exponents) are also considered. Chapter 14 introduces fractals, box dimension and Lyapunov dimension and iterated function systems.

The level of the material is suitable for advanced undergraduates or beginning graduate students, particularly those with some exposure to mathematical analysis and computer

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programming. The presentation is well-organized with some minor exceptions (e.g. the definition of a hyperbolic fixed point makes reference to the eigenvalues of linearized equations, while the process of linearization is not introduced until 13 pages later). The instructor who adopts this text may wish to place more emphasis on contrasting properties of discrete and continuous systems (e.g. different criteria for local asymptotic stability of fixed points, existence and uniqueness is a non-issue for discrete systems, chaos requires three or more dimensions for systems of differential equations, whereas, only one dimension is required for discrete systems, etc.). Students will find this text to be a stimulating introduction to the field of difference equations.

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