

Validating DEA as a Ranking Tool: An Application of DEA to Assess Performance in Higher Education.

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ABSTRACT. There is a general interest in ranking schemes applied to complex entities described by multiple attributes. Published rankings for universities are in great demand but are also highly controversial. We compare two classification and ranking schemes involving universities; one from a published report, “Top American Research Universities” by the University of Florida’s *TheCenter* and the other using DEA. Both approaches use the same data and model. We compare the two methods and discover important equivalences. We conclude that the critical aspect in classification and ranking is the model. This suggests that DEA is a suitable tool for these types of studies.

Key Words: Nonparametric Efficient Frontiers, Data Envelopment Analysis (DEA), Tiered Data Envelopment Analysis (TDEA), Linear Programming.

1. Introduction. Americans are voracious consumers of published evaluations and rankings of universities. Suppliers of these evaluations and rankings range from popular mass media such as *U.S. News and World Report* to specialized institutions such as *TheCenter* at the University of Florida. Different constituencies of the public have different interests and uses for these rankings. The magazine *U.S. News and World Report* evaluates and ranks universities using a set of attributes that target, ostensibly, prospective students and their parents; although the results are widely read, analyzed, and, if favorable, quoted by the institutions themselves. *TheCenter*, on the other hand, appeals to peer institutions, funding agencies, and other institutional consumers focusing on a different set of attributes.

Whatever the constituency, the product of these studies is some final classification with some sort of ranking. The practice of ranking universities or any collection of complex units characterized by multiple attributes is fraught with pitfalls and almost always controversial. The fact of the matter is that any such rankings involve arbitrary decision somewhere in the course of its creation. Even a more respected and scholarly study such as the one issued by the University of Florida requires several of these decisions.

The value and acceptance of a study and any resulting ranking is directly proportional to the experience and expertise of the people who produce it; the “experts”. The involvement of experts does not, however, guarantee recognition and approval. The ultimate validation of a ranking methodology is how well it fits the expectation of the target constituency especially at the top of the classification.

We will explore how data envelopment analysis (DEA) can be used for classifications and rankings. DEA is a nonparametric frontier estimation methodology originally introduced by Charnes *et al.* [1] in 1978 that compares functionally similar entities described by a common set of multiple numerical attributes. DEA classifies the entities into “efficient” or “performers” versus “inefficient” or “nonperformers.” The criterion for classification is determined by the location of the entities’ data point with respect to the efficient frontier of the production possibility set. The production possibility set is defined by the data and its shape depends on the assumption about the returns to scale which can be either constant (CRS), variable (VRS), increasing (IRS), or decreasing (DRS). The classification of any particular entity can be achieved by solving a linear program (LP). The term *model* as used in DEA and, in the present work, as it relates to any other type of analysis to compare, evaluate, and rank a collection of entities characterized by a common set of multiple, numerically valued, attributes refers exclusively to the composition of the attribute set.

In this work, we apply a DEA procedure to the model developed by *TheCenter* to evaluate the performance of American research universities and we compare our results with theirs. We study the scheme used by the *TheCenter*, contrast it conceptually to DEA and compare its results to a standard DEA-based approach. Our study demonstrates that DEA produces an assessment that matches closely the one produced by acknowledged experts. DEA, however, minimizes the amount of subjectivity needed for the analysis. This means that a more objective methodology can be applied to evaluate complex entities such as universities. The methodology itself serves to distinguish the subjective and objective aspects of the problem. Access to a study such as *TheCenter* is an opportunity to validate and promote DEA as a technique for classification and ranking.

2. Literature Review. This work will contrast and compare DEA with a classification and ranking study defined by experts in the area of higher education for assessing the performance of universities. The work fits into different categories. One is the vast body of work on presenting to the general public performance rankings of different entities such as the “best cities” studies of Fortune magazine or the many such studies applied to universities. Another, more specific, is the category of work on evaluating universities using DEA. A third category is the intersection of the first two, that is, studies that somehow contrast and compare popular rankings for general

public interest with results from a DEA study. In this section, we review selected studies from the categories of evaluating universities using DEA and works where DEA is applied on a model originally created for a different type of evaluation. We begin with the latter.

The works by Ahn, Arnold, Charnes, and Cooper [2], Breu and Raab [3], and Sarrico, Hogan, Dyson, and Athanassopoulos [4] stand out as examples where DEA is contrasted with other methods. Ahn, *et al.* [2] used the results of a study conducted by the Texas Select Committee on Higher Education (SCOHE) in 1986 using ratio efficiency and other management accounting approaches and compared it with DEA. Experts in the SCOHE studied higher education institutions in Texas and used five criteria as well as subjective factors to determine which schools should be closed and which should be merged. A DEA model loosely based on the SCOHE criteria was constructed. The DEA results agree with the SCOHE study in that all institutions selected for closure are indeed DEA inefficient and none of the efficient institutions are slated for closure. The issue of mergers does not present such a clear distinction. Of the mergers, some are between efficient and inefficient institutions, others are between two inefficient institutions, and there is a proposed merger between two efficient institutions. 75% of the DEA inefficient universities remain open. The results from DEA deviate substantially from those from SCOHE.

Breu and Raab [3] base a DEA model on the popular *U.S. News and World Report* ranking of national universities. This is one of the first papers to compare an independently created published ranking with results from a DEA study. The DEA study uses the same data but the model differs in that it incorporates only six of the twelve attributes used by the magazine. Another important disagreement between the two models is the assignment of attributes to the role of inputs; the DEA study selects four of its six performance attributes as inputs. This causes another discrepancy between the two models. The magazine considers all attributes to be performance “outputs” since they are assigned positive weights in the ranking calculation. The results reflect this different treatment of attributes. Only one in the top 15 universities is DEA efficient and six universities that are ranked 16 to 25 by the magazine turn out to be DEA efficient.

Sarrico, *et al.* [4] use data from the *Times Good University Guide* of UK universities for a DEA study. This study ranks universities based on a model with ten performance attributes. A feature of the DEA study is that it is custom-built for six different constituencies; each a category

of students. This generates six DEA models. The DEA models differ on the subsets of attributes used, on a preference structure for some of these attributes, and on the assignment of attributes as either inputs or outputs. Two of these DEA studies are of special interest since all ten attributes are used and treated as outputs. The results in these two cases are similar and there is an appearance of a strong correlation between the DEA rank based on the DEA score and the rank provided by the *Times*. Even though the use of DEA scores for ranking is problematic, the concordance between the ranks based on these scores and the ranks from the newspaper is excellent. The fact that these two specific DEA models are entirely output based (none of the ten original attributes are inputs) and that the results between the experts' ranking and the one arrived at using DEA closely agree suggests that DEA is a suitable methodology for performance evaluations using experts' model. The experts' data, model, and methodology validates DEA as a performance evaluation tool.

Another paper that compares results from a DEA study with a ranking published by a popular magazine is by Zhu [5]. Zhu used a DEA model based on attributes and data used in a rankings of cities based on thirteen diverse measures of quality of life published by *Fortune* magazine. The DEA results end up matching closely those obtained by the magazine despite the fact that the DEA model uses six of the attributes as inputs. This suggests that the magazine may have weighted these particular attributes as insignificant or, possibly, with a negative weight. This is not clear since there is no description of the methodology used by *Fortune* and the weight assignments were not published. It is hard to imagine a weighting scheme that rewards high values for attributes such as “Violent Crime Rate”.

A series of studies have appeared where DEA is used to study efficiency of universities with no connection to independent performance or ranking studies. In Ahn and Seiford [6], the efficiency of 153 U.S. higher learning institutions is determined using variable and constant returns to scale DEA formulations. Ahn, Charnes, and Cooper [7] used DEA to determine the efficiency of doctoral-granting institutions in the U.S. as defined by NCEES (National Center for Education Statistics) using data from HEGIS (Higher Education General Information Surveys).

Presumably, experts play an important role in the studies behind popular published rankings. Their input is valuable at several stages. In the modeling stage, the decision about the attribute mix requires knowledge, experience, and an understanding of the operation. The selection of

the methodology used to create the ranking also benefits from an expert decision. This almost always involves a decision about the assignment of weights to the different attributes. In ranking schemes such as *US News and World Report's* “America’s Best Colleges” or *Fortune's* “Best Cities”, each attribute receives a specific weight. These assignments are supposedly based on the experts’ knowledge and experience and acceptance by the intended audience will depend on the experts’ credibility and how well the results fit their perceptions. An analysis with DEA also requires an expert decision about the model. However, DEA can be said to have an advantage over other classification and ranking methodologies in that the weighting process can range between total objectivity and subjectivity. At one extreme there is no user input and weights are decided by the optimal solution to an LP. At the other extreme, weights for any or all attributes can be fixed at specific values in the LP formulation. As we will see next, a different weighting scheme is used by the University of Florida’s *TheCenter* for their “Top American Universities Research” Annual Report.

3. UFL’s *TheCenter* “Top American Universities Research” Annual Report. *TheCenter* is a research enterprise based at the university of Florida with a mission to understand, measure, and improve performance of American research universities. *TheCenter* produces the *Top American Universities Research* Annual Report which contains a classification and ranking of the research universities. The domain of the study is defined by all universities that have federal research expenditures as reported to NSF of at least \$20 million. The data used for this report are provided by peers throughout the country. The *Top American Universities Research* annual report is part of *The Lombardi Program on Measuring University Performance* [8] and has been issued every year for the past decade. Our results are based on the report issued in 2002 which involves 616 institutions.

The experts at *TheCenter* have developed a model and a specialized methodology to perform its classification and ranking of research universities. It is based on the placement in a ranking of a university with respect to one or more of a predefined set of magnitude attributes. The result is a classification with three main tiers. The first two tiers generate subtiers. Classification into one of the subtiers is determined by a “score” which is a count of the number of times a university appears in rankings with respect to nine attributes. The third tier is implied and there is no analysis for

it. It is composed of all the universities that do not appear in the top fifty of any of the original nine attributes used but appear in the top 200 list with an extra tenth attribute.

The attributes that are considered in the report are:

1. Total Research;
2. Federal Research;
3. National Academy Members;
4. Faculty Awards;
5. Doctorates Granted;
6. Postdoctoral Appointees;
7. Median SAT Scores;
8. Endowment Assets;
9. Annual Giving; and
10. National Merit and Achievement Scholars (only used in selecting third tier universities).

To be included in the top tier, a research institution must appear as a top 25 university in at least one of the first nine attributes in the list. A score is calculated by adding the number of times the university is ranked in the top 25. Harvard, MIT, and Stanford all have a score of 9, meaning that they are top 25 schools for all the attributes and they make up the top subtier within the first tier. The next subtier corresponds to universities that obtained a score of 8. This is the case of Columbia, Duke, Johns Hopkins, Berkeley, Pennsylvania, Michigan, and Minnesota. The third subtier is composed of universities that score 7; and so on. The last subtier includes universities with a presence in exactly one of the nine attributes. This main tier is made up of 52 institutions. For the complete classification go to *TheCenter’s Annual Report* [8]. The second main tier of universities is determined according to whether universities not in the first tier place between 26th and 50th in one or more of the nine attributes with scores calculated the same way as in the first tier. The second tier has a total of 32 institutions. The third and last tier contain all other universities that rank somewhere in the top 200 in at least one of the ten attributes for a total of 532 universities. No score is calculated for these universities so no subtiers are available at this level.

The classification and ranking scheme used by *TheCenter* assigns a magnitude data point in \mathbb{R}^{10} to a collection of entities selected using the initial inclusion criterion. Nine different rankings are performed, one for each of the first nine dimensions of the data vector. Each ranking assigns a specified number of universities to an exclusive tier. From within each tier, the scheme maps the

data point to another vector in the same space but with a much more restricted domain; i.e., into a vector of 0s and 1s. The mapping assigns a 1 at each dimension in which the entity was selected for inclusion to the tier. This mapping is then mapped again into a scalar value using the function that adds the total of 1s. This is the “score” assigned to the universities.

In order to implement the methodology used by *TheCenter* the following subjective decisions had to be made:

1. The domain of the study. The study involves only universities with more than \$20 million in annual federal research expenditures.
2. The model. The list of attributes used to make the measurement presumably involved experts who understand how a research university operates and how to measure its performance relative to the objective of the study.
3. The number of main tiers. The decision was to have three tiers. This is an arbitrary decision since any number of main tiers could have been defined.
4. The cut-off points for each main tier. Once the number of tiers has been decided, cut-off points can be generated a number ways. The current cut-off points are a presence in a ranking of top 25 universities in at least one of nine attributes; in the 26 to 50th position in the same nine attributes; and in the 51-200th position in a ranking with the original nine plus one more attribute.
5. Attribute hierarchy. The way the score is calculated, a presence at or above the cut-off point is counted once for each of the nine attributes. This implies that equal importance is given to all of the attributes.

This method is an instance of a more general scheme. The classification involves n entities selected according to initial inclusion criteria. Each entity is characterized by an m -dimensional magnitude vector, a^j ; $j = 1, \dots, n$. Each dimension of the data vector corresponds to an attribute in the model; so, a_i^j is the value of the i th attribute for entity j . Let $\mathcal{A} = \{a^1, \dots, a^n\}$ with $\mathcal{J} = \{1, \dots, n\}$ the index set. The index set is to be partitioned into K subsets $\mathcal{J}^1, \dots, \mathcal{J}^K$ which

will define the K principal tiers. Each of the m coordinates are used to determine inclusion to a tier based on specific $K - 1$ cut-off parameters t_i^k ; $k = 1, \dots, K - 1$; $i = 1, \dots, m$. The definition of t_i^k can be cardinal or ordinal; that is, they could determine a magnitude threshold or a position in a ranking. The tier inclusion procedure is as follows:

Procedure: "Generalized UFL Classification Scheme"

Initialization: $\mathcal{J}^k \leftarrow \emptyset$; $k = 1, \dots, K$.

For $k = 1$ to K :

For $i = 1$ to m :

Sort a_i^j ; $j = 1, \dots, n$, in decreasing order.

Let $\tilde{\mathcal{J}}^i$ be the permutation of \mathcal{J} that corresponds to the i th sorting, and $\tilde{\mathcal{J}}_j^i$; $j = 1, \dots, n$ its j -th element.

For $j = 1$ to n and $a^{\tilde{\mathcal{J}}_j^i} \notin \cup_{\ell=1}^k \mathcal{J}^\ell$

If $\begin{cases} a_i^{\tilde{\mathcal{J}}_j^i} \geq t_i^k & \text{for magnitude threshold;} \\ a_i^{\tilde{\mathcal{J}}_j^i} \succ t_i^k & \text{for ordinal threshold.} \end{cases}$

Then $\mathcal{J}^k \leftarrow a^{\tilde{\mathcal{J}}_j^i}$

Next j .

Next i .

Next k .

EndProcedure

Within each tier, the data vector, a^j , can be mapped to a 0-1 vector, δ^j , where

$$\delta_i^j = \begin{cases} 1 & \text{if } a_i^j \geq t_i^k \text{ or } a_i^j \succ t_i^k \text{ (depending on whether } t_i^k \text{ is a magnitude or ordinal threshold);} \\ 0 & \text{otherwise.} \end{cases}$$

In the general case, the final score for an entity is a function mapping the 0-1 vector, δ^j , to a scalar value where each component δ_i^j is multiplied by a weight, w_i^k ; $i = 1, \dots, m$; $k = 1, \dots, K$.

For the case of the UFL scheme we have:

Number of entities: $n = 616$

Number of attributes: $m = 9$ or $m = 10$, depending on tier.

Number of tiers: $K = 3$
 Threshold parameter (sorting position): $t_i^1 = 25, t_i^2 = 50, t_i^3 = 200; i = 1, \dots, m.$
 Weights for final score: $w_i^k = 1; k = 1, 2; i = 1, \dots, m.$
 (No scores are given for entities in the third tier.)

The system used by *TheCenter* for classifying and ranking complex entities using a few basic tiers is reminiscent of other discrete tier assessment systems such as those based on “stars” for classifying hotels and restaurants. The similarities are more than superficial. Hotels and restaurants are complex entities that need to be assessed using multiple attributes related to service quality, infrastructure, and facilities. Many subjective decisions are involved but eventually the assessment for a star system will have to define thresholds and a weighting scheme. Another important similarity is their functionality for their intended audience. The star system is a useful assessment guide for travelers and diners since it provides immediate comprehensible information about the performance of the entity. The star system, like the system used by *TheCenter*, acknowledges that it would be unfair to the entities and a disservice to the users to give the appearance of greater discrimination by providing a numerical score for each entity based on direct weighting of the attributes. It is clear that the discrete tier system for classifying and ranking complex entities is a natural, intuitive, and useful approach that is widely recognized and used. In the following section we will see how such a system can be interpreted within the DEA framework.

4. Equivalence Between UFL’s Classification and Ranking Method and DEA. The tiering classification methodology used by *TheCenter* can be interpreted within the context of DEA.

Consider the following definition for points in \mathfrak{R}^9 based on *TheCenter*’s model and data:

$$\delta^j = \begin{bmatrix} \delta_1^j \\ \vdots \\ \delta_9^j \end{bmatrix};$$

where

$$\delta_i^j = \begin{cases} 1, & \text{attribute } i \text{ for university } j \text{ is above the threshold for the corresponding tier;} \\ 0, & \text{otherwise.} \end{cases}$$

Universities in the first two tiers generate a vector δ^j . For example, Harvard’s vector is $(1, \dots, 1)^T$ while that for Wisconsin is $(1, 1, 1, 1, 0, 1, 1, 1, 0)^T$. Consider the VRS production possibility set generated by all the vectors δ^j within one tier. This is the unbounded polyhedral set in \Re^9 defined by the four axioms of Banker, *et al.* [9], when the data are the vectors δ^j .

We will show that the UFL’s *TheCenter* scoring and subtier classification scheme is equivalent to applying a DEA VRS analysis to the vectors δ^j .

Consider the following LP where the vector j^* is one of the data vectors in the data domain of a DEA study:

$$\begin{aligned} \max_{\lambda, \mathcal{S} \geq 0} \quad & e\mathcal{S} \\ \text{s.t.} \quad & \sum_j \delta^j \lambda_j - I\mathcal{S} \geq \delta^{j^*} \\ & \sum_j \lambda_j = 1 \end{aligned} \tag{VRS - 1}$$

This is the *additive* LP formulation of Charnes, *et al.* [10] the solution to which provides a necessary and sufficient condition for efficiency classification of entity j^* . The entity j^* is on the VRS efficient frontier if and only if the LP’s objective function value is 0. The next result will use this LP to show how the secondary tier assignments can be arrived at using a VRS efficiency classification. We assume that the data set is *complete* in the following sense: if the vector with the most 1s has \hat{k} of these, then any of the combinations of 0s and 1s where the number of 1s is less than or equal to \hat{k} can be found in the rest of the vectors. For example, if $\hat{k} = m$, then the data set is composed of all 2^m combinations of 0s and 1s. Note that without loss of generality, we also assume that there is no duplication of vectors.

RESULT. *Let DMU \hat{j} have \hat{k} 1s with no other DMUs having more 1s. Then DMU \hat{j} is VRS-efficient and no other DMU with fewer 1s is efficient.*

PROOF. See Appendix 1.

The result states that if the data set is complete and without duplicates and includes the vector $(1, \dots, 1)$, this vector is the only one on the efficient frontier. Moreover, if this vector is omitted from the data set then the DEA analysis will identify the remaining m vectors with $m - 1$ 1s as VRS efficient. These correspond to the entities with the second highest score within a tier in the

UFL scheme. And if these are removed all vectors with $m - 2$ 1s are efficient and correspond to third highest scores; and so on.

An important observation about this equivalence is the use of “peeling” away (i.e., removing) efficient vectors from the data set and repeating the analysis on what remains. Different levels of efficiency are obtained when efficient DMUs are removed from the data to uncover the next “layer” of efficient DMUs. The idea of peeling in DEA has been proposed before. Barr *et al.* [11], use it to provide a classification for inefficient DMUs. The idea has a strong intuitive appeal and a direct counterpart is used in point-depth nonparametric statistical analyses (see, e.g., Barnett [12] or Liu [13]). We will employ this technique in our application of DEA to the UFL’s *TheCenter* data in the next section.

5. DEA Analysis vs UFL’s *TheCenter* Method. In this section, we demonstrate how DEA performs as a classification and ranking tool by comparing results from a DEA study to the ones obtained by the experts at *TheCenter*.

Comparisons of results between two studies are fair if the models involved are identical to each other and if they are based on the same data set. For this study, we used the set of nine attributes that was used by *TheCenter*. The data provided by *TheCenter* and available to the public, however, contained some missing values. In fact, a total of 10% of the values were missing for “*Endowment Assets*,” “*Annual Giving*,” and “*Median SAT*”. Missing values can be problematic in DEA. For this reason, we imputed the missing values using regression. A total of 614 universities were included in the data set. Two universities, namely, Louisiana State University - Shreveport and University of Colorado - Colorado Springs were removed from the original data set provided by *TheCenter* because values associated with a key attribute (“*Federal Research*”) were missing.

DEA was not originally intended as a ranking technique. However, if used properly, DEA has a role to play as a ranking tool. Barr *et al.* introduced the idea of “peeling the DEA onion” [11]. As we have seen, this intuitive approach squares well with aspects of the method used by UFL. The peeling approach classifies DMUs into groups based on demonstrated levels of achievement. DEA is first applied to the entire data set to find the efficient DMUs that will compose the first tier. Efficient DMUs are then removed from the data set and another DEA analysis is applied to this

truncated data set. The next group of efficient DMUs compose the second tier and is also removed from the data set. DMUs in this second tier are not efficient compared to the DMUs in the first tier but are efficient compared to all the others. This process continues until all the DMUs in the data set have been assigned to a tier. The complete DEA analysis required 34 runs.

The DEA application is based on a model where all the attributes are output performance levels. This makes the DEA returns to scale assumption de facto VRS (Caporaletti *et al.* [14]). We used DEA Frontier, a solver tool add-in to Microsoft Excel developed by Joe Zhu [15].

Table 1 exhibits the results of the first four DEA runs, i.e., the universities composing tiers 1, 2, 3, and 4. For the complete DEA layered classification, refer to Appendix 3.

The information in Table 2 is organized to help compare the two results. The first column, “UFL’s Classification: First Three Subtiers”, is a complete listing of the universities that are classified by the UFL as belonging to the first three subtiers; i.e., they obtained a score of 7, 8, or 9 in the first main tier. In the second column, “DEA Classification: Matching Universities”, each university from the first column is listed along side so that tier classifications can be compared. Additionally, below the horizontal rule in column 2 we complete the list of Tiers 1 and 2 universities that were excluded in the matching above the rule. Note that although three Tier 3 universities appear in the matchings in the second column above the rule, this is not the complete list of Tier 3 universities.

Tier	DEA University Tier Classification with UFL's <i>TheCenter</i> Model and Data
1	California Institute of Technology Harvard University Johns Hopkins University Massachusetts Institute of Technology Stanford University University of California - Berkeley University of Michigan - Ann Arbor
2	Columbia University Princeton University University of California - Los Angeles University of California - San Diego University of Illinois - Urbana Champaign University of Pennsylvania University of Texas - Austin University of Washington - Seattle University of Wisconsin - Madison Yale University Yeshiva University
3	Cornell University Dartmouth College Duke University Emory University Harvey Mudd College Northwestern University Rice University Texas A&M University University of California - San Francisco University of Chicago University of Colorado - Boulder University of Minnesota - Twin Cities University of North Carolina - Chapel Hill Washington University
4	Baylor College of Medecine Brown University Carnegie Melon University Case Western Reserve University Georgia Institute of Technology Ohio State University - Columbus Pennsylvania State University - University Park Pomona College Rockefeller University Swarthmore College University of Arizona University of California - Davis University of Florida University of Notre Dame University of Pittsburgh - Pittsburgh University of Southern California University of Virginia

Table 1. DEA Results (continued in Appendix 3).

UFL’s Classification: First Three Subtiers.	Subtier	DEA Classification: Matching Universities.	Tier
Harvard	1	Harvard	1
M.I.T.	1	M.I.T.	1
Stanford	1	Stanford	1
Columbia	2	Columbia	2
Duke	2	Duke	3
Johns Hopkins	2	Johns Hopkins	1
U. of Cal. - Berkeley	2	U. of Cal. - Berkeley	1
U. of Pennsylvania	2	U. of Pennsylvania	2
U. of Michigan	2	U. of Michigan	1
U. of Minnesota	2	U. of Minnesota	3
Cornell	3	Cornell	3
Yale	3	Yale	2
U. of Cal. - Los Angeles	3	U. of Cal. - Los Angeles	2
U. of Washington - Seattle	3	U. of Washington - Seattle	2
U. of Wisconsin - Madison	3	U. of Wisconsin - Madison	2
(For Remaining Tier 1 classification refer to Appendix 2.)		Remaining Tier 1 & 2 Universities:	
		Cal. Tech.	1
		Princeton	2
		U. of Cal. - San Diego	2
		U. of Illinois	2
		U. of Texas - Austin	2
		Yeshiva	2
		(For Remaining Tier 3 go to Table 1.)	

Table 2. UFL’s vs. DEA classifications: DEA ordered to lign-up with UFL’s ordering.

We can observe that the DEA results match closely the classification obtained by *TheCenter*. The UFL’s Subtier 1 universities (Harvard, M.I.T. and Stanford) are a proper subset of the first DEA tier. In fact, 100% of the universities in the first 3 subtiers of the UFL ranking appear in the first 3 subtiers of the results obtained with DEA. There is not, however, an indication that the method used by UFL is particularly discriminating since its first three tiers compose nearly one half (47%) of the universities in the top three DEA tiers. In fact, we may argue that the presence of CalTech in Tier 1 and Princeton, UC San Diego, Illinois and Texas at Austin in Tier 2 in the DEA classifications conforms well to our expectations. This consistency of results is the validation we seek for DEA as a tool for classification and ranking.

These results indicate that the critical aspect in classification and ranking is the model. Table 2 supports this argument for the case of the top tier entities where it counts most. The strong correspondence between UFL's classification for the top three subtiers and DEA's results for its top three tiers means that UFL's methodology, despite its many subjective decisions, is basically determined by the choice of attributes. This provides direction as to how to focus efforts when planning a rankings study; clearly, the model is the driving factor. Our results also provide a compelling argument for using DEA once an appropriate model has been constructed.

6. Conclusion. This article validates DEA as a classification and ranking tool. This validation is lacking and as a consequence we do not see DEA used for these purposes as much as it should be. Our approach was to study a respected and accepted classification scheme; namely, UFL's *theCenter*, "Top American Research Universities" and establish connections with DEA. We then used their model and data to run a tiered DEA analysis in the spirit of Barr, Durchholz, and Seiford [10]. The results obtained using DEA agreed for the most part with the ones from *TheCenter's* report. They also produced results that match well our perceptions about how these universities should be classified.

An important conclusion is that a critical aspect in classification and ranking is the selection of attributes; i.e., the model. This was the only aspect of the study that was common to the two methods; yet, the results are strikingly similar, especially at the top levels in the classification. However, in contrast with the method used by UFL, DEA is entirely objective once the model has been constructed. This gives DEA a clear advantage as an analytical tool as long as the appropriate attention is given to building the model. It is our hope that these prescriptions and advantages will be recognized by analysts and their publishers and we will start to see more published classification and ranking reports in the mass media that use DEA as the main methodology.

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Appendix 1.

Proof from Section 4

RESULT. Let DMU \hat{j} have \hat{k} 1s with no other DMUs having more 1s. Then DMU \hat{j} is VRS-efficient and no other DMU with fewer 1s is efficient.

PROOF. Suppose we are scoring a DMU, \tilde{j} , with fewer than \hat{k} 1s using the VRS Additive LP (*VRS* – 1) from Section 4. We can construct directly a feasible solution to the LP by identifying a DMU with \hat{k} 1s in the data set such that all its 1s are in the same row as DMU \tilde{j} . This DMU must exist by our completeness assumption. At every row where this DMU has a 1 and where the right hand side is zero we use the appropriate slack to cancel it out. The objective function value of this solution is equal to the number of slacks used for the cancelation. This means that there is a feasible solution with $z > 0$. Since the LP is a maximization, the optimal objective function value, z^* , whatever it may be, is $z^* \geq \hat{z} = 1$ and therefore clearly not zero. We can conclude that the right-hand side is an inefficient DMU.

Suppose DMU \hat{j} has \hat{k} 1s and no other DMU has more. The location of the 1s in DMU \hat{j} are in rows $\hat{i}_1, \dots, \hat{i}_k$. Let \bar{A} be the matrix of coefficients of the system where DMU \hat{j} has been deleted (as well as all other DMUs with more 1s than \hat{k}). Then if we let $\pi_{\hat{i}_1} = 1$, $\beta = -(\hat{k} - 1) + \epsilon$ for some small enough ϵ , we have a feasible solution to the alternate Farkas system establishing by Farkas’ Lemma the infeasibility of the deleted domain (*VRS* – 1) system. This implies not just efficiency but extreme efficiency. ■

Appendix 2.

UFL, *TheCenter*

Rank	<i>TheCenter</i> – Top 25 Research Universities
1	Harvard University Massachusetts Institute of Technology Stanford University
2	Columbia University Duke University Johns Hopkins University University of California - Berkeley University of Pennsylvania University of Michigan - Ann Arbor University of Minnesota - Twin Cities
3	Cornell University Yale University University of California - Los Angeles University of Washington - Seattle University of Wisconsin - Madison
4	University of Southern California Washington University University of California - San Francisco
5	University of North Carolina - Chapel Hill Princeton University University of California - San Diego University of Illinois - Urbana Champaign
6	University of Chicago Pennsylvania State University - University Park University of Texas - Austin
7	Northwestern University Ohio State University - Columbus Texas A&M University University of Arizona University of Florida University of Virginia California Institute of Technology Emory University
8	University of Pittsburgh - Pittsburgh University of California - Davis Baylor College of Medicine Dartmouth College Michigan State University Rice University
9	Case Western Reserve University University of Colorado - Boulder Purdue University - West Lafayette Rutgers the State University of NJ - New Brunswick University of Maryland - College Park University of Texas SW Medical Center - Dallas Vanderbilt University Brown University Rockefeller University University of Notre Dame Indiana University - Bloomington Yeshiva University Indiana University-Purdue University - Indianapolis

TheCenter's Results.

Appendix 3. DEA as a ranking tool - Results

Tier	University Rankings with DEA as a Tool
5	Amherst College Baylor College of Medicine Georgetown University Michigan State University Middlebury College New York University North Carolina State University Nova Southeastern University Purdue University - West Lafayette Tufts University University at Stony Brook University of California - Santa Barbara University of Iowa University of Maryland - College Park University of Rochester University of Texas SW Medical Center - Dallas Vanderbilt University Williams College
6	Boston College Boston University Brandeis University Carleton College Cooper Union for the Advancement of Science & Art Indiana University - Bloomington Indiana University-Purdue University - Indianapolis Mount Sinai School of Medicine Rutgers the State University of NJ - New Brunswick Thomas Jefferson University University at Buffalo University of Alabama - Birmingham University of California - Irvine University of Cincinnati - Cincinnati University of Delaware University of Georgia University of Utah Wake Forest University Wellesley College Wesleyan University
7	Arizona State University - Tempe Bowdoin College College of William and Mary George Washington University Grinnell College Haverford College Iowa State University Louisiana State University - Baton Rouge Oregon Health & Science University Rensselaer Polytechnic Institute Smith College Southern Methodist University Texas Tech University Tulane University University of California - Santa Cruz University of Colorado Health Sciences Center University of Connecticut - Storrs University of Illinois - Chicago University of Kansas - Lawrence University of Kentucky University of Maryland - Baltimore University of Massachusetts - Amherst University of Miami University of Missouri - Columbia University of Nebraska - Lincoln University of Richmond University of Texas MD Anderson Cancer Center Vassar College Virginia Polytechnic Institute and State University

 Tier University Rankings with DEA as a Tool

- 8 Brigham Young University - Provo
 City University of New York - City College
 Clemson University
 Colgate University
 Colorado State University
 Illinois Institute of Technology
 Lehigh University
 Macalester College
 Oberlin College
 Oregon State University
 Reed College
 Rose-Hulman Institute of Technology
 Saint Louis University - St. Louis
 Syracuse University
 Texas Christian University
 University of California - Riverside
 University of Hawaii - Manoa
 University of Houston - University Park
 University of Louisville
 University of Massachusetts Medical Sch - Worcester
 University of Massachusetts Medical Sch - Worcester
 University of New Mexico - Albuquerque
 University of Oklahoma - Norman
 University of Oregon
 University of South Carolina - Columbia
 University of South Florida
 University of Tennessee - Knoxville
 University of Texas Health Science Center - Houston
 University of Texas Medical Branch - Galveston
 Washington State University - Pullman
 Wayne State University
-
- 9 Auburn University - Auburn
 Barnard College
 Bates College
 Bryn Mawr College
 City University of NY - Graduate Sch and University Ctr
 Colby College
 Colorado School of Mines
 Drexel University
 Howard University
 Kansas State University
 Loyola University Chicago
 Medical University of South Carolina
 Mississippi State University
 Mount Holyoke College
 Northeastern University
 Oklahoma State University - Stillwater
 Polytechnic University
 Temple University
 Trinity University
 University at Albany
 University of Alabama - Tuscaloosa
 University of Arkansas - Fayetteville
 University of Connecticut - Health Center
 University of Texas Health Science Ctr - San Antonio
 University of Tulsa
 University of Vermont
 US Naval Postgraduate School
 Virginia Commonwealth University
 Woods Hole Oceanographic Institution
-

Tier University Rankings with DEA as a Tool

- 10 Binghamton University
Bucknell University
California Polytechnic State Univ - San Luis Obispo
Clark University (MA)
Fordham University
Franklin & Marshall College
George Mason University
Georgia State University
Kettering University
Lafayette College
Marquette University
MCP Hahnemann University
Medical College of Wisconsin
Meharry Medical College
New Mexico State University - Las Cruces
Pennsylvania State University - Hershey Medical Ctr
Rush University
San Diego State University
Stevens Institute of Technology
Teachers College at Columbia University
Trinity College (CT)
Uniformed Services University of the Health Sciences
University of Akron - Akron
University of Denver
University of Maine - Orono
University of Mississippi - Oxford
University of Missouri - Kansas City
University of Missouri - Rolla
University of Nebraska Medical Center
University of Nevada - Reno
University of New Hampshire - Durham
University of North Texas
University of Rhode Island
University of Tennessee Health Science Center
University of Wisconsin - Milwaukee
University of Wyoming
Utah State University
West Virginia University
Whitman College
Worcester Polytechnic Institute
-

Tier	University Rankings with DEA as a Tool
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- | | |
|----|--|
| 11 | Allegheny College |
| | American University |
| | Catholic University of America |
| | Charles R. Drew University of Medicine and Science |
| | Clark Atlanta University |
| | Clarkson University |
| | Colorado College |
| | Desert Research Institute |
| | Duquesne University |
| | Florida Atlantic University |
| | Florida International University |
| | Hamilton College (NY) |
| | Louisiana State University Health Sciences Center |
| | Medical College of Georgia |
| | Miami University - Oxford |
| | Michigan Technological University |
| | Montana State University - Bozeman |
| | Morehouse School of Medicine |
| | New School University |
| | North Dakota State University |
| | Northern Illinois University |
| | Occidental College |
| | Ohio University - Athens |
| | Rochester Institute of Technology |
| | San Francisco State University |
| | Santa Clara University |
| | Southern Illinois University - Carbondale |
| | Texas A&M University - Corpus Christi |
| | Union College (NY) |
| | Union Institute |
| | University of Alaska - Fairbanks |
| | University of Arkansas for Medical Sciences |
| | University of Central Florida |
| | University of Dayton |
| | University of Idaho |
| | University of Kansas Medical Center |
| | University of Maryland - Baltimore County |
| | University of Massachusetts - Boston |
| | University of Massachusetts - Lowell |
| | University of Montana - Missoula |
| | University of New Orleans |
| | University of North Dakota |
| | University of Oklahoma Health Sciences Center |
| | University of Southern Mississippi |
| | University of Texas - Arlington |
| | University of Texas - Dallas |
| | US Naval Academy |
| | Western Washington University |
-

Tier	University Rankings with DEA as a Tool
12	California State University - Fullerton California State University - Long Beach City University of New York - College of Staten Island Cleveland State University College of the Holy Cross Connecticut College Creighton University Denison University East Carolina University East Tennessee State University Florida A&M University Fuller Theological Seminary in California Furman University Illinois State University Kent State University - Kent Loma Linda University Manhattan College Marshall University Medical College of Ohio New Jersey Institute of Technology New York Medical College OHSU - Oregon Graduate Institute Sch of Sci & Eng Ponce School of Medicine Portland State University Rutgers the State University of NJ - Newark San Jose State University Skidmore College South Dakota School of Mines and Technology Spelman College State Univ. of New York Health Science Ctr - Brooklyn Texas A&M University System Health Sciences Center University of Alabama - Huntsville University of Arkansas - Little Rock University of Colorado - Denver University of Louisiana - Lafayette University of Memphis University of Nevada - Las Vegas University of North Carolina - Greensboro University of Northern Colorado University of Puerto Rico - Mayaguez University of Puerto Rico - Medical Sciences University of South Alabama - Mobile University of Texas - San Antonio University of the Pacific University of Toledo US Air Force Academy Valparaiso University Villanova University Wright State University - Dayton

Tier	University Rankings with DEA as a Tool
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- | | |
|----|---|
| 13 | Albany Medical College |
| | Ball State University |
| | Bowling Green State University - Bowling Green |
| | Bradley University |
| | California School of Professional Psych - San Diego |
| | California State University - Northridge |
| | City University of New York - Queens College |
| | Eastern Virginia Medical School |
| | Fairfield University |
| | Finch University of Health Science - Chicago Med School |
| | Georgia Southern University |
| | Grand Valley State University |
| | Gustavus Adolphus College |
| | Hampshire College |
| | Hofstra University |
| | Jackson State University |
| | John Carroll University |
| | Long Island University (Multiple campuses) |
| | New Mexico Institute of Mining and Technology |
| | North Carolina A&T State University |
| | Northern Arizona University |
| | Oakland University |
| | Old Dominion University |
| | Southwest Missouri State University |
| | St. Olaf College |
| | State Univ. of New York - Coll of Enviro Sci and Forestry |
| | State Univ. of New York Upstate Medical University |
| | Tennessee State University |
| | University of Maryland Biotechnology Institute |
| | University of Maryland Center for Environmental Science |
| | University of Missouri - St. Louis |
| | University of North Texas Health Science Ctr - Fort Worth |
| | University of Puerto Rico - Rio Piedras |
| | University of Texas - El Paso |
| | Virginia Military Institute |
| | Western Michigan University |
-

Tier	University Rankings with DEA as a Tool
14	California School of Professional Psych - Alameda California State University - Dominguez Hills California State University - Fresno Calvin College Central Connecticut State University Central Michigan University Central Washington University Chapman University City University of New York - Brooklyn College City University of New York - Hunter College Claremont Graduate University College of Charleston College of Wooster DePaul University Dickinson College Eastern Michigan University Fitchburg State College Florida Institute of Technology Hampton University Institute of Paper Science and Technology Ithaca College Knox College Lewis & Clark College Loyola College in Maryland Mercer University - Macon New Mexico Highlands University New York Institute of Technology (Multiple campuses) Pace University (Multiple campuses) Pitzer College Radford University Rutgers the State University of NJ - Camden Seton Hall University Simmons College South Dakota State University Southern Illinois University - Edwardsville Southern Oregon University Southern University and A&M College State Univ. of New York - College at Buffalo State Univ. of New York - College at Purchase Texas A&M University - Commerce Towson University University of Dallas University of Mississippi Medical Center University of Nebraska - Omaha University of North Carolina - Charlotte University of Northern Iowa University of San Francisco University of South Dakota University of Southern Maine University of Wisconsin - Whitewater US Coast Guard Academy Western Kentucky University Wichita State University Willamette University

 Tier University Rankings with DEA as a Tool

- 15 Alabama A&M University
 Alcorn State University
 Alfred University
 Alcorn State University
 Alfred University
 Appalachian State University
 Baylor College of Dentistry
 Boise State University
 Bridgewater State College
 California School of Professional Psych - Fresno
 California State University - Bakersfield
 California State University - Chico
 California State University - Los Angeles
 California State University - San Bernardino
 City University of New York - Bernard M Baruch College
 Drake University
 Eastern Kentucky University
 Eastern Washington University
 Evergreen State College
 Fairleigh Dickinson University
 Fort Lewis College
 Humboldt State University
 Idaho State University
 Indiana State University
 Indiana University of Pennsylvania - Indiana
 James Madison University
 Kirksville College of Osteopathic Medicine
 La Salle University
 Lamar University - Beaumont
 McNeese State University
 Milwaukee School of Engineering
 Minnesota State University - Mankato
 Mississippi Valley State University
 Morehouse College
 Morgan State University
 Murray State University
 Norfolk State University
 North Carolina Central University
 Northern Kentucky University
 Ohio Wesleyan University
 Pacific University
 Pittsburg State University
 Point Loma Nazarene University
 Prairie View A&M University
 Providence College
 Southeast Missouri State University
 Southwest Texas State University
 St. Mary's College of Maryland
 St. Mary's University
 State Univ. of New York - College at Fredonia
 State Univ. of New York - College at Geneseo
 State Univ. of New York - College of Optometry
 Stephen F. Austin State University
 Tennessee Technological University
 Texas A&M University - Kingsville
 Texas Woman's University
 Truman State University
 Tuskegee University
 University of Detroit Mercy
 University of Hartford
 University of Houston - Clear Lake
 University of North Carolina - Wilmington
 University of San Diego
 University of Texas - Pan American
 University of Wisconsin - Green Bay
 University of Wisconsin - La Crosse
 University of Wisconsin - Stout
 West Chester University of Pennsylvania
 West Virginia State College
 Western Illinois University
 Widener University - Chester
 Xavier University of Louisiana
-

Tier	University Rankings with DEA as a Tool
16	Adelphi University Air Force Institute of Technology Delaware State University Fort Lewis College Hope College Saint Joseph’s University South Carolina State University Spalding University St. John’s University (NY) University of Louisiana - Monroe University of West Florida
17	Abilene Christian University Albany College of Pharmacy Andrews University Antioch University (Multiple campuses) Gallaudet University Louisiana Tech University Midwestern University (IL) Oral Roberts University Texas Southern University Universidad Central Del Caribe University of Guam University of Massachusetts - Dartmouth University of the Sciences in Philadelphia Wentworth Institute of Technology Wesleyan College
18	Augsburg College Embry-Riddle Aeronautical University - Daytona Beach (FL) Grambling State University Juniata College Middle Tennessee State University Sam Houston State University University of Alaska - Anchorage University of Arkansas - Pine Bluff University of Maryland - Eastern Shore University of North Carolina - Asheville University of Portland University of Wisconsin - Eau Claire Virginia State University
19	Arkansas State University - Jonesboro Fayetteville State University Fisk University Lake Forest College Langston University Montana Tech of the University of Montana Northeastern Ohio Universities College of Medicine Pennsylvania College of Optometry Western Carolina University Winston-Salem State University

Tier	University Rankings with DEA as a Tool
20	Lincoln University (MO) Maharishi University of Management Massachusetts College of Pharmacy and Health Sciences Philadelphia College of Osteopathic Medicine State Univ. of New York - College at Oswego Tarleton State University University of New Haven University of the Virgin Islands Youngstown State University
21	Bowie State University Lincoln University (MO) University of the District of Columbia
22	California State Polytechnic University - Pomona Institute of Textile Technology Le Moyne-Owen College
23	Albany State University Benedict College City University of New York - Lehman College City University of New York - York College Elizabeth City State University Monmouth University St. Cloud State University State Univ. of New York - College at Old Westbury University of Central Arkansas West Texas A&M University
24	Alabama State University Chicago State University New England College of Optometry
25	Bennett College Coastal Carolina University Kentucky State University Savannah State University State University of West Georgia Tougaloo College University of Houston - Downtown University of Illinois - Springfield University of Wisconsin - Stevens Point
26	Bethune-Cookman College City University of New York - John Jay College Criminal Justice Oakwood College Plattsburgh State University Southeastern Louisiana University University of Health Sciences University of Tennessee - Chattanooga Virginia Union University

Tier	University Rankings with DEA as a Tool
27	Central State University Lawrence Technological University Rider University Salem International University State Univ. of New York - College at Brockport Sul Ross State University - Alpine University of Southern Colorado Western University of Health Sciences
28	City University of New York - Medgar Evers College Claffin University East Stroudsburg University of Pennsylvania Kennesaw State University University of Findlay University of Wisconsin - Oshkosh
29	Des Moines University - Osteopathic Medical Center Johnson C. Smith University Southern Connecticut State University University of Central Oklahoma University of Wisconsin - Parkside Wilberforce University
30	Arkansas Tech University Dillard University Nicholls State University Rust College State Univ. of New York - College at Cortland University of Wisconsin - Superior
31	Montclair State University Morris Brown College Regis University Shaw University Southern University - New Orleans University of Alaska - Southeast University of Missouri Systems Office Western State College of Colorado
32	Fairmont State College Jarvis Christian College Kutztown University of Pennsylvania Northeastern Illinois University Sonoma State University University of Nebraska - Kearney University of Wisconsin - River Falls West Virginia School of Osteopathic Medicine
33	Pontifical Catholic University of Puerto Rico - Ponce Southern College of Optometry
34	Ferris State University Philander Smith College