Supplementary material for the article “*Accurate and efficient power calculations for 2 × m tables in unmatched case-control designs*” by József Bukszár and Edwin J. C. G. van den Oord

The power of Pearson's statistic is plotted against the critical value for 2 × 2 tables in Figure 1, and 2 × 3 tables in Figure 2 by red circles, which overlap forming a red line. We chose to display the region for which power was greater than 0.5 because in practice most power calculations are in this range. We calculated the exact power of Pearson's statistic for the small 2 × 2 tables in Figure 1. Since calculating the exact power of Pearson's statistic for the large 2 × 3 tables in Figure 2 is not tractable, simulation with 5,000,000 replications was used. The approximations AE, CE and CA are plotted by black, green and blue lines, respectively.

The accuracy of both AE and CE are good, in fact, both the green and the black line are close to the red one. In contrast for CA, the difference is large at some points. For example, for several critical values the difference is over 0.1. In addition, the inaccuracy of CA is ***unsystematic***; it underestimates power in Fig 1a and Fig 2a, and overestimates the power in Fig 1b and Fig 2b. Furthermore, Fig 2a shows that CA may be inaccurate even when the effect size is small.

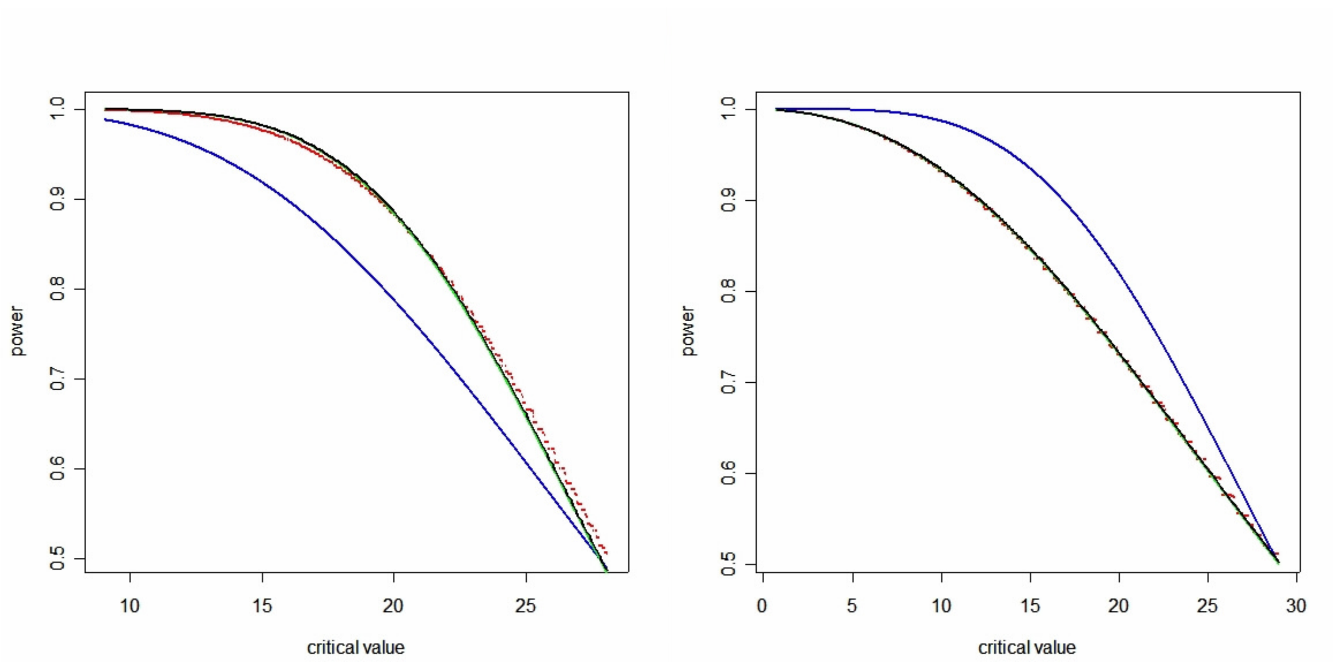

Figure 1.a.-b.: The exact power of Pearson's statistic as well as the power approximated by AE, CE and CA are plotted against the critical value for 2 × 2 table with p_1 = 0.1, q_1 = 0.5, np = 50, nq = 300 (fig. a) and for 2 × 2 table with p_1 = 0.1, q_1 = 0.5, np = 400, nq = 20 (fig. b). The exact power is plotted by red the red line; the blue, the green and the black line represent the power approximation obtained by CA, CE and AE, respectively.

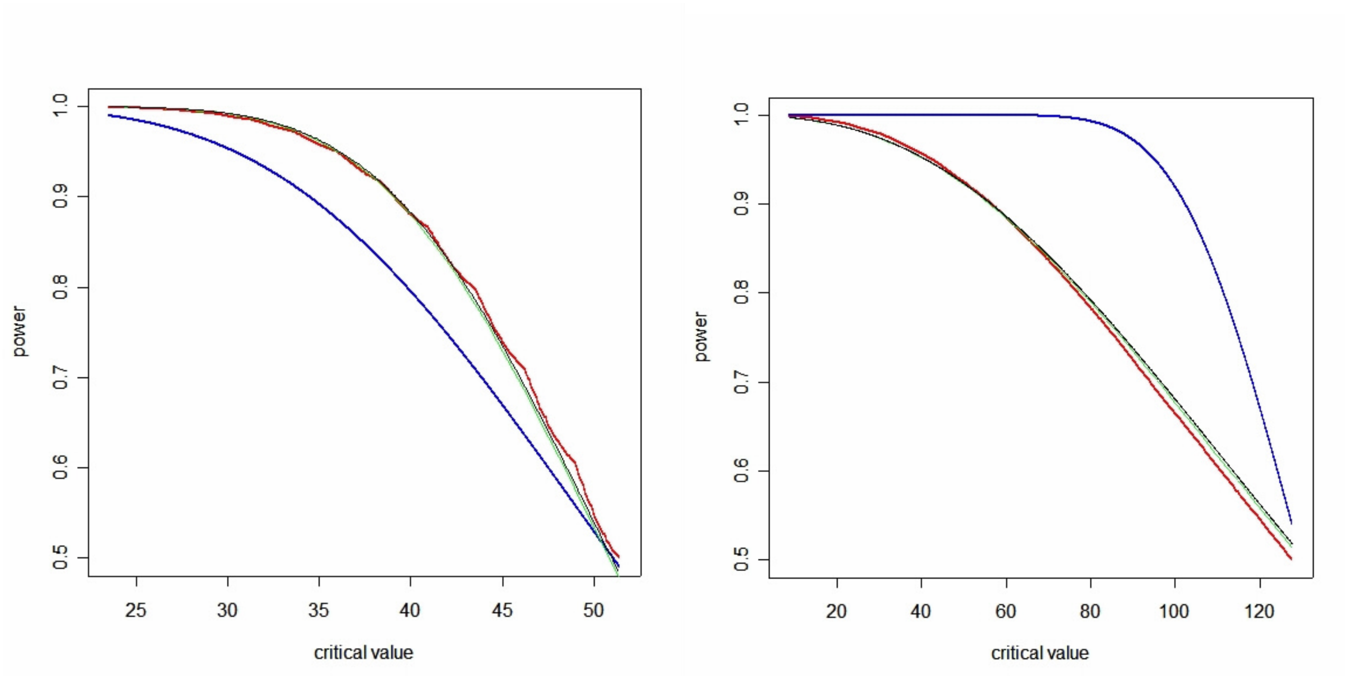

Figure 2.a.-b.: The exact power of Pearson's statistic as well as the power approximated by AE, CE and CA are plotted against the critical value for 2 × 3 table with p_1 = 0.1, p_2 = 0.3, q_1 = 0.8, q_2 = 0.1, *np* = 50, *nq* = 50 (fig. a) and for 2 × 3 table with p_1 = 0.46, p_2 = 0.41, q_1 = 0.49, q_2 = 0.5, *np* = 100, *nq* = 9900 (fig. b). The exact power is plotted by red the red line; the blue, the green and the black line represent the power approximation obtained by CA, CE and AE, respectively.

To study the accuracy of approximations CE, CA and AE for 2 × 2 tables with large sample size, numerical results are summarized in Table 1-3 (see the file tables.pdf). The tables report the *Mean of the Absolute Differences (MAD)* between the power calculated by the exact method and by the approximation at the non-continuity points of the "power function" where exact power is over 0.5. The lower the MAD value, the better is the approximation. In Tables 1-3, we fixed p_1 to 0.05 while ranging q_1 from 0.1 to 0.9 to study different effect sizes. In all conditions in Table 1, 2 and 3 the total sample size is 500, 1,000 and 5,000, respectively. Results show that, except the first row in Table 1 where the smallest expected cell frequency is less than 5 (particularly 2.5), the MAD of AE and CE remains zero in the first two decimals. Note that the accuracy of CE and AE is not affected by ratio *p* or by the effect size, furthermore, it gets better when the total sample size increases. The CA is clearly the poorest approximation. The MAD of CA is higher than 0.04 in almost half of the conditions studied and sometimes even exceeds 0.1 (e.g. when q_1 = 0.5, *np* = 100 and *nq* = 900). In general, the inaccuracy of CA increases when the effect size increases. However, the first column in every table shows that CA may be inaccurate also for small effect sizes, particularly when *p* deviates substantially from .5. The accuracy of the CA does not improve when the total sample size increases.

Table 1: Table reports mean absolute differences (MAD) between the approximations provided by the methods CE, CA, AE and the power calculated by the exact method in the range where power > 0.5. The values of q_1 are listed on the top and the values of np and nq are listed on the left-hand side, $p_1 = .05$ in all examples.

$np, nq \setminus q_1$		.1	.3	.5	.7	.9
50	CE	0.0632	0.0581	0.0102	0.0358	0.0278
450	CA	0.0630	0.0643	0.1146	0.1188	0.0620
	AE	0.0632	0.0581	0.0099	0.0339	0.0252
250	CE	0.0030	0.0035	0.0033	0.0037	0.0056
250	CA	0.0068	0.0329	0.0470	0.0619	0.1091
	AE	0.0029	0.0034	0.0027	0.0029	0.0054
450	CE	0.0078	0.0022	0.0036	0.0050	0.0050
50	CA	0.0199	0.0513	0.0481	0.0325	0.0130
	AE	0.0078	0.0022	0.0035	0.0048	0.0049

Table 2: Table reports mean absolute differences (MAD) between the approximations provided by the methods CE, CA, AE and the power calculated by the exact method in the range where power > 0.5. The values of q_1 are listed on the top and the values of np and nq are listed on the left-hand side, $p_1 = .05$ in all examples.

$np, nq \setminus q_1$		.1	.3	.5	.7	.9
100	CE	0.0067	0.0062	0.0069	0.0052	0.0027
900	CA	0.0135	0.0323	0.1135	0.0921	0.0375
	AE	0.0067	0.0062	0.0066	0.0038	0.0023
500	CE	0.0020	0.0023	0.0024	0.0030	0.0033
500	CA	0.0062	0.0327	0.0467	0.0628	0.1104
	AE	0.0019	0.0022	0.0019	0.0020	0.0028
900	CE	0.0046	0.0013	0.0022	0.0024	0.0028
100	CA	0.0267	0.0496	0.0459	0.0296	0.0097
	AE	0.0046	0.0013	0.0021	0.0023	0.0030

Table 3: Table reports mean absolute differences (MAD) between the approximations provided by the methods CE, CA, AE and the power calculated by the exact method in the range where power > 0.5. The values of q_1 are listed on the top and the values of np and nq are listed on the left-hand side, $p_1 = .05$ in all examples.

$np, nq \setminus q_1$		.1	.3	.5	.7	.9
500 4500	CE	0.0046	0.0040	0.0041	0.0026	0.0020
	CA	0.0415	0.1039	0.1101	0.0866	0.0358
	AE	0.0046	0.0039	0.0038	0.0018	0.0018
2500 2500	CE	0.0008	0.0011	0.0011	0.0013	0.0015
	CA	0.0057	0.0314	0.0480	0.0619	0.1084
	AE	0.0008	0.0010	0.0011	0.0014	0.0015
4500 500	CE	0.0021	0.0004	0.0018	0.0015	0.0017
	CA	0.0263	0.0467	0.0440	0.0273	0.0105
	AE	0.0021	0.0004	0.0017	0.0014	0.0019