Predicting shape and stability of air–water interface on superhydrophobic surfaces with randomly distributed, dissimilar posts

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A mathematical framework developed to calculate the shape of the air–water interface and predict the stability of a microfabricated superhydrophobic surface with randomly distributed posts of dissimilar diameters and heights is presented. Using the Young–Laplace equation, a second-order partial differential equation is derived and solved numerically to obtain the shape of the interface, and to predict the critical hydrostatic pressure at which the superhydrophobicity vanishes in a submersed surface. Two examples are given for demonstration of the method's capabilities and accuracy. © *2011 American Institute of Physics*. [doi:10.1063/1.3590268]

A combination of hydrophobicity and microfabricated roughness can result in a phenomenon known as superhydrophobicity. The drag force imparted by a moving liquid in contact with a superhydrophobic surface is reduced because the air entrapped inside the pores of the surface reduces the contact between water and solid walls.[1] Superhydrophobic surfaces can, therefore, be exploited to reduce the drag force exerted on submerged moving objects such as ships, submarines, or torpedoes. Superhydrophobic surfaces are often manufactured by microfabrication of grooves or posts on a hydrophobic surface. When the pore space on a superhydrophobic surface is filled with air, the system is considered to be at the Cassie state.[2] If the hydrostatic pressure is high, water may penetrate into the pores of the surface and replace the air. This results in the elimination of superhydrophobicity, and transition to the so called Wenzel state.[3,4] The pressure at which a superhydrophobic surface departs from the Cassie state, and therefore the superhydrophobic property vanishes, is referred to as the critical pressure in this work.

Balance of forces has previously been used to study the meniscus shape in capillary tubes,[5,6] capillary channels,[7] shape of a droplet,[8–12] and liquid bridge that forms when a solid disk is withdrawn from a liquid reservoir,[13] as well as the capillary rise between vertical plates.[14] Only a few studies have applied balance of forces to investigate the shape and stability of the air–water menisci of superhydrophobic surfaces. Extrand[15,16] applied balance of forces to a superhydrophobic surface with ordered pillars to study the stability of the meniscus formed by a drop on the surface. By applying balance of forces, an analytical relationship was proposed by Zheng *et al.*[17] to predict the stability of the meniscus formed between the cylindrical posts or square pillars on a microfabricated superhydrophobic surface.

In the present work, we developed a method to calculate the meniscus shape under different hydrostatic pressures of any microfabricated superhydrophobic surface with randomly distributed circular posts of dissimilar diameters, heights, and materials. The shape of the meniscus is then used to calculate the critical hydrostatic pressure at which the superhydrophobicity of the surface vanishes. Note that the existing models can only be applied to surfaces with orderly distributed identical posts. Beyond the microfabricated superhydrophobic surfaces, the present method can potentially be used to calculate the critical pressure of surfaces with fibrous superhydrophobic coatings.[18,19]

Consider a superhydrophobic surface with a set of identical posts orderly placed next to one another. Figure 1(a) shows a schematic of a post and the corresponding meniscus; d, h, and L represent the post diameter, height, and the center to center distance between two neighboring posts, respectively. Note that because of the geometrical symmetry, only

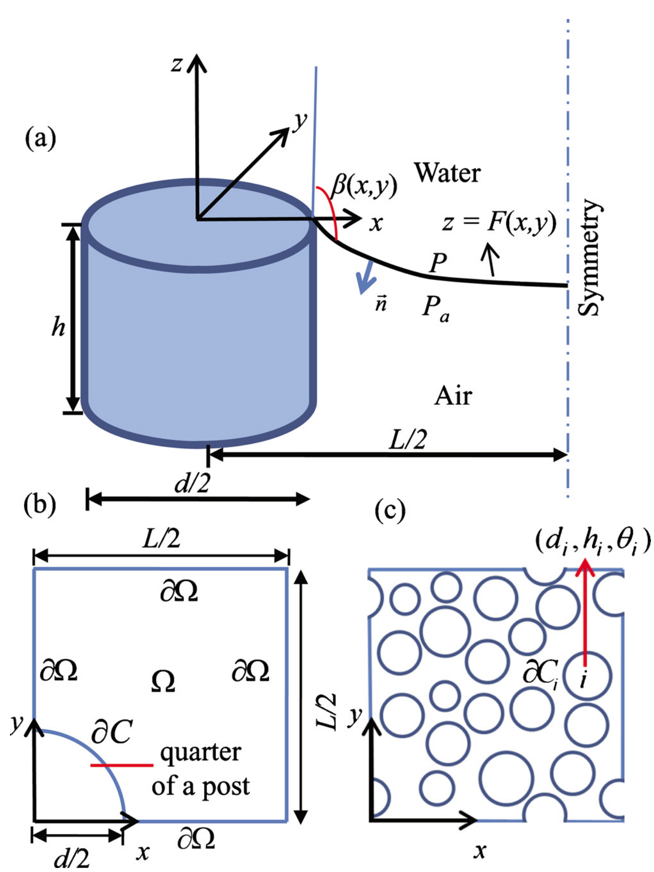

FIG. 1. (Color online) Schematic of (a) a post with the corresponding air–water meniscus; (b) computational domain for a surface with ordered identical posts; and (c) computational domain for a general surface with randomly distributed dissimilar posts.

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