Math 307: Multivariable Calculus
SAMPLE FINAL EXAM

[*Note*: For the actual open-book Exam as with this one, *it will be assumed that you will have your book with you together with a functioning calculator.* Please note that no calculators or books other than your own will be provided for the test. The questions below are intended to give some idea about the type and number of questions on the Final Exam. To be fully prepared, additional practice with the examples and exercises in the book is necessary.]

Instructions. Write clean and for full credit, provide details and computations, as well as correct notation and diagram labels.

1. Find all the second partial derivatives of the function

$$f(x, y, z) = 3z\cos(x^2 - y).$$

2. Determine the equation of the tangent plane to the surface $xy+yz+zx = 3$ at the point $(1, 2, 1/3)$. Simplify your answer as much as possible.

3. If $z = \cos(xy) - x\cos y$ where $x = u - v^2 + 3$ and $y = u^2 + v - 1$, use the chain rule to compute the partial derivatives $\partial z/\partial u$ and $\partial z/\partial v$.

4. Use the method of Lagrange multipliers to compute the maximum and minimum values of the function $f(x, y, z) = 4x - 2y - 2z + 3$ subject to the constraint equation $x^2 + y^2 + z^2 = 6$.

5. Calculate the following iterated integral by properly reversing the order of integration:

$$\int_0^1 \int_{y^2}^1 y\cos(x^2)\,dxdy$$

6. (a) Compute the following iterated integral by first converting it to polar coordinates (a is an unspecified positive constant):

$$I = \int_0^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \frac{2}{\sqrt{4 + x^2 + y^2}}\,dxdy.$$

(b) Determine a positive value of a so that $I = \pi a$.

7. A lamina occupies the region D bounded by the parabola $y = 1 - x^2$ and the x-axis. If the mass density $\rho(x, y)$ of the lamina at each point is proportional to the distance from the x-axis, find the center of mass of this lamina.

8. Determine the volume of a solid region that lies under the paraboloid $z = 6 - x^2 - 2y^2$ and above the rectangle $R = [0, 2] \times [0, 1]$.

9. Compute the triple integral

$$\iiint_E 12y \, dV$$

where E is the solid region below the surface $z = 1 + x - y^2$ and above the square $[0, 1] \times [0, 1]$.

10. Compute the work done by the force field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} - 2z\mathbf{k}$ on an object that moves in space along

(a) The helical path $\mathbf{r}(t) = \sin(\pi t)\mathbf{i} + \cos(\pi t)\mathbf{j} + t\mathbf{k}$ from $t = 0$ to $t = 1$.

(b) The straight line $\mathbf{r}(t) = (1 - t)\mathbf{i} + (t - 2)\mathbf{j} + t\mathbf{k}$ from $t = 0$ to $t = 1$.

11. The acceleration of a particle moving in space is given by the formula

$$\mathbf{a}(t) = \mathbf{i} + \frac{1}{(t+1)^2}\mathbf{j}.$$

(a) If at time $t = 0$, the particle was at the origin and moving with velocity $\mathbf{v}(0) = \langle 1, -1, 1 \rangle$, compute the position or the trajectory of the particle $\mathbf{r}(t)$ at any time t.

(b) At time $t = 1$, determine the velocity, speed and location of the particle as well as the curvature of the trajectory $\mathbf{r}(t)$.

12. The temperature is a region of space is given as $T(x, y, z) = e^{2x-y+z}$.

(a) Compute the rate of change (or the directional derivative) of the temperature at the point $(1, 2, 0)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{k}$.

(b) Determine the maximum rate of change of the temperature at the point $(1, 2, 0)$ and the direction in space in which this maximal rate of change occurs.