Impossibility of Existence of Opposite Equilibrium Preferences on Law Enforcement

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Abstract

Law enforcement is among the key elements of a civil society that ensures the achievement of a higher social welfare. An enforcement authority determines the level of public enforcement of law. In this paper, I show that it is impossible to have agents with "opposite equilibrium preferences" over the level of public enforcement of law in the same society.

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1 Introduction

Since Becker [3] has introduced an economic analysis of law enforcement, there has been a vast literature on economics of crime and punishment.[1] In public enforcement of law models, it is assumed that there is an enforcement authority who determines the level of enforcement for harmful acts in the society. Law enforcement monitors the activities of agents in the society. When law enforcement detects an agent engaged in a harmful activity, that agent is fined. As the level of law enforcement increases, the likelihood of

[1] For a survey of the literature, see Polinsky and Shavell [8]

agents being caught who engage in harmful activities increases. In the literature, different objectives for the enforcement authority have been proposed. In Becker [3], the objective of the enforcement authority is to minimize social cost. The objective of the enforcement authority in Polinsky and Shavell [6] is to maximize the total expected utility of agents in the society. Cooter and Ulen [4] (p. 510) propose that the aim of the enforcement authority should be to minimize social cost, which has different components from the social cost in Becker [3].

In this paper I consider an alternative approach to determine the level of public enforcement of law: A social choice rule, which maps preferences of agents to a socially desirable outcome, is used to choose the level of enforcement. In this setting, each agent reports his preference on law enforcement to a central authority. Then, this central authority using a predetermined social choice rule selects a socially optimal outcome. In order to apply a social choice rule in a law enforcement environment, the first step I take is to look at the structure of agents' preferences over enforcement levels in *enforcement equilibrium*. In enforcement equilibrium, given the level of enforcement, agents will decide whether they will engage in the harmful activity or not. An agent's utility from that level of enforcement depends not only on his own action but also on actions of other agents. This is because other agents' actions determine the harm he faces, and affect fine revenues used to finance public enforcement along with taxes collected. A close observation of equilibrium preferences reveals that the domain of profiles of equilibrium preferences are restricted.

I show that marginal equilibrium expected utilities of any two agents are identical for all enforcement levels except those at which one of those agents engages in the activity whereas the other does not. This result implies that in any society it is impossible to have agents with *opposite equilibrium preferences*: In equilibrium, if there is an agent who prefers more enforcement to less, then there cannot be another agent in the same society who prefers less enforcement to more. Another implication of this result is that the domain of profiles of equilibrium preferences over enforcement levels does not contain the class of profiles of *single-peaked equilibrium preferences*. More specifically, the only possible profiles of single-peaked equilibrium preferences over enforcement levels consist of *monotone equilibrium preferences*. Moreover, if all agents in the society have monotone equilibrium preferences, then their equilibrium preferences are either all monotone increasing or all monotone decreasing in enforcement level.

In Becker [3] the objective of social planner is to minimize social cost, which consists of harms caused by agents, costs of detection (enforcement), and the costs of punishment to the criminals less gains of criminals. Agents and their choices are implicitly explained, and they get utility from consumption, and face uncertainty because of the possibility of getting caught. On the other hand, Cooter and Ulen [4] (p. 510) also suggest that the aim of the law maker is to minimize social cost but it consists of costs of protection and the net harm caused, i.e. social loss, while the crime is committed. Becker [3] shows that the optimal probability of detecting the crime is the lowest possible subject to the highest fine that can be paid by offenders. His result is based on the risk neutrality of agents. Polinsky and Shavell [6] study risk averse agents as well as risk neutral agents. In their model, if an agent does not engage in the harmful activity, then his consumption consists of his wealth remaining from taxes and insurance premium which covers all losses due to harms caused by agents' actions in the society. If he engages in the activity, then his consumption, in addition to taxes and insurance premium paid, will increase with his gain from the activity. If he gets caught, then his consumption will decrease with fine paid. Harms due to harmful activities are felt equally likely by everyone in the society. If agents are risk neutral, then the premium paid can also be seen as the risk they bear due to potential harms caused by others in the society. The objective of the enforcement authority in Polinsky and Shavell [6] is to maximize the total expected utility of agents in the society. They show that if agents are risk neutral, then it is optimal to set the probability of detection to a minimum level below which it is not possible to detect any crime, and the punishment level as high as possible which is constrained by wealth of agents. They have two results when agents are risk averse. The first one is that the probability of detection should be set to 1 when agents are risk averse and the cost of monitoring is sufficiently small. The second one is that it may not be optimal to set the enforcement level low and the punishment level high even if the cost of monitoring is very large when agents are risk averse. They say that these results are more realistic compared to the results of the risk neutral case.

In Polinsky and Shavell [7], the objective of the law maker is to maximize total expected utility of agents in the society. The expected utility of an agent consists of his gain from engaging in the harmful activity, expected loss due to the possibility of getting caught, expected loss due to the possibility of being a victim of a crime, and the per capita cost of enforcement. If the agent does not engage in the harmful activity, then the expected utility will not include the gain from engaging in harmful the activity, and the expected punishment. They show that in equilibrium, expected punishment for the harmful activity will always be less than the harm it causes.

2 The Model

There is a unit measure of agents in the society. Each agent $i \in [0,1]$ has the option of either engaging in the activity which causes harm in the amount of $e > 0$ and deriving utility of $v_i > 0$, or not engaging in the harmful activity and getting 0 utility. $g(\cdot)$ is an integrable probability density function representing the distribution of values of agents in the society. Enforcement agency monitors the actions of agents and detects agents engaged in the harmful activity with probability $p \in [0,1]$. If an agent is detected, he pays a fine of $f > 0$. In this paper, I assume that the fine level f is fixed and does not exceed any agent's budget but it is high enough such that for each agent there is a probability of detection p at and above which he will not engage in the harmful activity, i.e., $\frac{\overline{v}}{f} < 1$ where $\overline{v}$ is the highest possible valuation in the society. Enforcement at level p costs $c(p) > 0$ where $c(.)$ is assumed to be a differentiable function. Revenues from the fines collected at enforcement level p and the cost $c(p)$ of detection at probability p are assumed to be equally shared by all agents in the society. Given the probability of detection p and fine level f, each agent i engages in the harmful activity as long as $v_i \geq pf$. Harmful action is denoted by a_i, and the default action of not engaging in the harmful activity is denoted by a_i^0. Given the probability of detection p and fine level f, an *enforcement equilibrium* is a set of actions $\{a_i^*\}_{i\in[0,1]}$ such that for each agent i action $a_i^* \in \{a_i^0, a_i\}$ maximizes his utility.

Given fine level f, for each agent i his *equilibrium expected utility* [2] at p under risk neutrality is

$$Eu_i^*(p) = v_i(a_i^*) - pf(a_i^*) - e\int_{pf}^{\overline{v}} g(v)\mathrm{d}v - c(p) + pf\int_{pf}^{\overline{v}} g(v)\mathrm{d}v.$$

The expression $pf\int_{pf}^{\overline{v}} g(v)\mathrm{d}v$ is the revenue from fines collected at enforcement level p. On the other hand, the expression $e\int_{pf}^{\overline{v}} g(v)\mathrm{d}v$ has two possible interpretations: If the harm is a *public harm*, meaning all agents are affected by each harmful activity, then $e\int_{pf}^{\overline{v}} g(v)\mathrm{d}v$ is the total harm in the society caused by agents engaged in the harmful activity. If the harm is

[2] I would like to thank Andy McLennan for suggesting this term.

a *private harm*, meaning a random agent is offended by each harmful activity, and each agent is equally likely to be affected by each harmful activity, then $e \int\limits_{pf}^{\overline{v}} g(v)\mathrm{d}v$ is the expected total harm each agent faces caused by agents engaged in the harmful activity.[3]

3 Results

The Theorem below shows that for any two agents, their marginal equilibrium expected utilities are identical at enforcement levels where they make the same decision about engaging in harmful activity.

Theorem. *For each agent $i, j \in [0,1]$ such that $v_i < v_j$, and for each level of enforcement $p \in [0, \frac{v_i}{f}) \cup (\frac{v_j}{f}, \frac{\overline{v}}{f}]$, marginal equilibrium expected utilities of agents i and j are equal, i.e.*

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} = \frac{\mathrm{d}Eu_j^*(p)}{\mathrm{d}p},$$

and for each $p \in (\frac{v_i}{f}, \frac{v_j}{f})$

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} > \frac{\mathrm{d}Eu_j^*(p)}{\mathrm{d}p}.$$

Proof. Observe that for each agent $i \in [0,1]$, his marginal equilibrium utility at each enforcement level $p \in [0, \frac{\overline{v}}{f}]$ such that i engages in the harmful activity, i.e. $pf < v_i$, is

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} = f[(e - pf)g(pf) - G(pf)] - c'(p), \tag{1}$$

and his marginal equilibrium utility at each enforcement level $p \in [0, \frac{\overline{v}}{f}]$ such that i does not engage in the harmful activity, i.e. $pf > v_i$, is

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} = f[(e - pf)g(pf) - G(pf) + 1] - c'(p). \tag{2}$$

Hence, if $p \in [0, \frac{v_i}{f}) \cup (\frac{v_j}{f}, \frac{\overline{v}}{f}]$, then agents i and j have identical marginal equilibrium expected utilities, i.e.

[3] For more on private versus public harms see, for example, Bator [1], Head [5], and Baumol and Oates [2].

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} = \frac{\mathrm{d}Eu_j^*(p)}{\mathrm{d}p}.$$

Observe that by equations 1 and 2, if $p \in (\frac{v_i}{f}, \frac{v_j}{f})$, then

$$\frac{\mathrm{d}Eu_i^*(p)}{\mathrm{d}p} > \frac{\mathrm{d}Eu_j^*(p)}{\mathrm{d}p}.$$

□

The Theorem implies that for each agent $i, j \in [0,1]$ such that $v_i < v_j$, and at each enforcement level $p \in [0, \frac{v_i}{f}) \cup (\frac{v_j}{f}, \frac{\overline{v}}{f}]$, equilibrium expected utilities of i and j are either both weakly increasing or both weakly decreasing in p. An immediate consequence of the Theorem, Corollary 1 below shows that there do not exist two agents such that when one always prefers more enforcement to less, the other always prefers less enforcement to more. Hence, the impossibility of existence of opposite equilibrium preferences on law enforcement emerges.

Corollary 1. *There do not exist $i, j \in [0,1]$ such that i's equilibrium expected utility is monotone decreasing in enforcement level p and j's equilibrium expected utility is monotone decreasing in enforcement level p.*

A common assumption on preferences in social choice theory is *single-peakedness*. An agent is said to have a *single-peaked equilibrium preferences* if there exists an enforcement level p_i^* such that for all enforcement levels p and p' such that $p < p'$ if $p' \leq p_i^*$, then $Eu_i^*(p) < Eu_i^*(p')$, and if $p \geq p_i^*$, then $Eu_i^*(p) > Eu_i^*(p')$. If the set of equilibrium expected utilities are restricted to be single-peaked, then the following Corollary to the Theorem holds.

Corollary 2. *If all agents have single-peaked equilibrium expected utilities over enforcement level, then either all agents have equilibrium expected utilities monotone increasing in enforcement level $p \in [0, \frac{\overline{v}}{f}]$, or all agents have equilibrium expected utilities monotone decreasing in enforcement level $p \in [0, \frac{\overline{v}}{f}]$.*

As a result of Corollary 2, if the single-peakedness of equilibrium expected utilities are assumed, then there exists a unique unanimously agreed policy, and it is either no enforcement $p = 0$, or the full enforcement $p = \frac{\overline{v}}{f}$. Also, this policy is *strategy-proof*, i.e. it is in the best interest of all agents in the society to report their true preferences. If equilibrium preferences are not restricted, then there are two cases under which all agents agree on the enforcement level. These two cases are stated in the following Corollary to the Theorem.

Corollary 3. *If the harm level e is sufficiently high, then all agents have an equilibrium expected utility increasing in enforcement level p. If the harm level e is sufficiently low, cost of detection is monotone increasing in detection level, and the lowest valuation in the society is positive, then all agents have an equilibrium expected utility decreasing in enforcement level p.*

The proof of the first part of Corollary 3 follows from equation 1. Let $\underline{v} > 0$ denote the lowest valuation in the society. Observe that for $e < \underline{v}$

$$(e - pf)g(pf) < 1 - G(pf).$$

Hence, by equation 2 the second part of Corollary 3 holds.

4 Conclusion

In this paper I study the structure of preferences of agents over levels of law enforcement in a standard enforcement model. The model has interesting implications on preferences of agents: Even though it may not seem unreasonable to have two agents in the same society with completely opposite preferences on law enforcement, the main result of the paper shows that this is not possible at all.

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