Memories of Ron Graham

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My connection to Ron begins with Joe Spencer who was my Master's advisor at Stony Brook. In my first year, 1985-6, János Pach and Vera Sós were both Visitors, Paul Erdős stopped by, and Béla Bollobás came up from LSU. I guess that was my introduction to the Hungarian Mathia. I think Joel ran the Colloquium that year, because it was a non-stop parade of Combinatorics giants: Herb Wilf, Dan Kleitman, Curtis Greene, Richard Stanley, ..., and Ron Graham. How could anyone resist such a lure? In my memory of things, Ron gave his talk with a wry smile and a twinkle in his eye, like he was hiding a secret. I would come to appreciate that facial expression for years to come. That summer I read Graham/Rothschild/Spencer (Ramsey Theory) on the beach, attracting all sorts of comments from my family ("he's learning how to invite people to a party where everyone knows each other").

When Joel moved to NYU, he suggested that I consider Rutgers, to work with Ron, who had just become affiliated with Rutgers because of the birth of DIMACS. I first met Ron in his new and completely barren office at Rutgers. He opened the desk drawer and was surprised to see two small balls, which he began to bounce on the desk. He said he had heard about some balls that run out of bounces after a while. Sure enough, one of them landed like a thud, with no bounce. I immediately realized that I was in for some fun, and I'd better stay on my toes.

Typically, Ron and I met about every two weeks, at Bells Labs rather than Rutgers, and my 30-40-minute drive from New Brunswick to Murray Hill was filled with anticipation, wondering what circus-like environment I would encounter next. Usually, "circus-like" is metaphorical but, with Ron, it was sometimes literal. One time I walked into his office and there were coins of every type scattered all over the floor, desk, chairs, shelves, etc. I asked what happened and he said that Penn & Teller just left, and that he was teaching them a new trick (which involved balancing a coin on the tip of a coat hanger and spinning the hanger around your finger without the coin falling off.) For a long time, we tried to see how many times you could bounce a pen off the wall in one throw. We could get three hits pretty consistently, but never four. Once he came to work very excited to tell me that one of the physicists at the Labs proved that, with a perfect cylinder and frictionless surface, four was impossible. I said, "okay, give me a perfect cylinder and frictionless surface" — then off we went to look for imbalanced pens with some grip. One of our more semi-dangerous adventures involved throwing a lacrosse ball down three floors in the center of one of their stairwells. Normal throwing spins the ball, forcing it to bounce toward anyone walking up the stairs, while it is difficult to throw a knuckle ball hard enough to return to the third floor. Ron was just insatiably curious about everything all the time. This was the kind of stuff that used to get me into so much trouble in high school.

But we did some math, too. With the breadth of interests and sheer numbers of people coming and going at Bell Labs, Rutgers, and DIMACS, I felt lucky to be exposed every nook and cranny of discrete math. Of course, you could ask Ron anything, and he'd likely know who has the latest result and what the open questions were. Thus it was difficult to think about which of all these wonderful directions to go in. I did have some success with a question of Ed Scheinerman on the dimension of circle containment orders that resulted in my first paper. But then Ron got me hooked on universal cycles, which he and Fan Chung and Persi Diaconis were just getting off the ground, and which became my thesis topic. That was exciting to be a part of, since it was new and everything was open. But it was also challenging because it was new, and techniques were few. Interestingly, that's one topic Ron and I rarely discussed, outside of me informing him of my progress, or lack thereof. He wanted to make sure that people knew that what I produced was mine,

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and not his. It certainly made my results more self-satisfying, and probably did help land my first job at Arizona State (although I think Tom Trotter, who hired me, was likely more interested in my poset result!).

I will mention that I did get good at the hanger trick, and also learned the universal cycle card trick that Persi invented, which one can read about in their book ([2]). I felt that, in one's thesis defense, one should exhibit what they learned from their advisor that they might not have been able to learn elsewhere, and so I performed both. I guess that's the one thing I could do better than Ron — of all the crazy, impossible-looking things he could do, he never really learned the perfect shuffle.

It was also in those years that graph pebbling was born. I remember sitting with Fan, Dan Kleitman, and Dan Ullman, having lunch in a little café near George Washington University during the 1989 Capital City Conference on Combinatorics, while Fan described this fascinating little puzzle on the n-cube. We went through lots of napkins that day, to no avail. Of course, Fan solved it a few weeks later and wrote the first paper on the topic ([1]). Contained in it was an unassuming little gem: Graham's Conjecture.[1] 30 years later, we're still plugging away at it.

I'm grateful for learning site-swap juggling from Ron. It's a great mental and physical diversion, of course, but it's also a sneaky way to introduce young people to some very lovely mathematics, and to the notion that mathematics is everywhere, if only you will look. I'm satisfied to have discovered two universal cycle-related juggling patterns, 631415241 and 531441335224512, and learned to perform them. It's even more fulfilling, though, to have gone into K-12 classrooms and shared these things with young, eager minds. Juggling, card tricks, integer sequences, sorting networks, secret codes, puzzles, combinatorial games, tilings, Rubik's cubes, etc. I've been doing this for over 20 years (we call it *Crazy Math Day*), inspired by Ron's inviting, infectious, and encouraging manner, his constant sharing of interesting, curious, and surprising things, and his ability to present the simplest, irresistibly intriguing question that hides within it a larger theory. So, there's a sense in which I've introduced Ron to thousands of kids, who hopefully now associate Math with Fun.

As so many can attest to, Ron's concern for people went well beyond mathematics. He was well known for putting people first, ahead of professional and institutional concerns. He certainly shared professional and administrative advice with me throughout my career and, more importantly, touched the lives of my wife and family in meaningful ways, including having invited my son to the Gathering for Gardner conference because of his Rubik's Cube interest. Since Ron's passing they have learned to unicycle in his honor.

I imagine that he and Uncle Paul are thumbing through *The Book* these days, smiling about the beautiful proofs the rest of us have yet to discover. Have fun, Ron. Know that we miss you.

References

[1] F.R.K. Chung, *Pebbling in hypercubes*, SIAM J. Discrete Math. **2** (1989), no. 4, 467–472.

[2] P. Diaconis and R. Graham, Magical Mathematics: The Mathematical Ideas That Animate Great Magic Tricks, Princeton University Press, Princeton, NJ, 2012.

[1] The pebbling number of the cartesian product of two graphs is at most the product of the individual pebbling numbers.