Derivation of formula to calculate loss of stable RNA and mRNA

Definitions:

$\mathbf{S}_n$ = amount of stable RNA in sample n
$\mathbf{M}_n$ = amount of mRNA in sample n
$\mathbf{S}_n'$ = amount of stable RNA per cell in sample n
$\mathbf{M}_n'$ = amount of mRNA per cell in sample n
$\mathbf{C}_n$ = number of cells in sample n
f_s = factor relating $\mathbf{S}_1'$ to $\mathbf{S}_2'$ $\quad (\mathbf{S}_1' * f_s = \mathbf{S}_2')$
f_m = factor relating $\mathbf{M}_1'$ to $\mathbf{M}_2'$ $\quad (\mathbf{M}_1' * f_m = \mathbf{M}_2')$
$\mathbf{R} = \mathbf{M}_1' / \mathbf{S}_1'$

1. Equal RNA is loaded (more or less), giving:

1a. $S_1 + M_1 = S_2 + M_2$
1b. $(S_1' * C_1) + (M_1' * C_1) = (S_2' * C_2) + (M_2' * C_2)$

2. The ratio of total fluorescent intensity in the experimental condition (condition 2) and that of the control condition (condition 1) is:

2a. $M_2/M_1 = (S_1 + M_1 - S_2) / M_1$
$= (S_1' C_1 + M_1' C_1 - S_1' f_s C_2) / (M_1' C_1)$ [definition]
$= (S_1' + M_1' - S_1' f_s C_2/C_1) / M_1'$
$= (1 + R - f_s C_2/C_1) / R$
2b. $C_2/C_1 = (S_1' + M_1') / (S_2' + M_2')$ [1b]
$= (S_1' + M_1') / (S_1' f_s + M_1' f_m)$
$= (1 + R) / (f_s + R f_m)$
2c. $M_2/M_1 = (f_s + R f_m + R f_s + R^2 f_m - f_s - R f_s) / (R f_s + R^2 f_m)$
$= (f_m + R f_m) / (f_s + R f_m)$

3. R is very small. 2% in *E. coli* growing under normal conditions, 5% for *E. coli* growing very slowly (1.5 hr doubling)

3a. $M_2/M_1 \approx f_m / f_s$ [2c, $\mathbf{R} \approx 0$]

4. f_s can be calculated, given M_2/M_1 and f_m

M_2/M_1 is measurable, as the ratio of total signal in the experimental condition to the total signal in the control condition (no normalization).
f_m is measurable as the same ratio but after normalization

4a. $f_s = f_m [(1 + R) / (M_2/M_1) - R]$ [2c]
4b. $\approx f_m / (M_2/M_1)$ [3a]

The calculated value of f_s is not very sensitive to **R**. If **R** is as high as 50% (which would be pretty remarkable), then the error in the calculation of f_s is only:

$(1 - \mathbf{M_2/M_1}) / (2\, \mathbf{M_2/M_1}) = 13\%$ for the most extreme case of $\mathbf{M_2/M_1} = 1.34$