

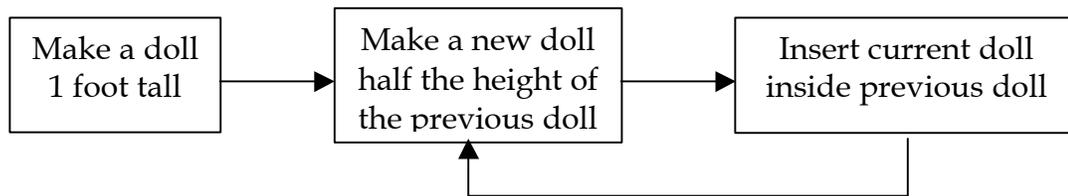
## MATH 195: Gödel, Escher, and Bach (Spring 2000)

### Problem Set 6: *Recursive Structures and Processes*

To be discussed Tuesday, February 20

These problems range from the relatively straightforward to rather difficult. Don't get frustrated if you're stumped by one or another, but go as far as you can on them by Tuesday.

- 6.1.** I have a Russian doll inside a doll inside a doll (the Russian word is "matryoshka"). Mine is rather special in its construction. The factory used the following procedure:



The process goes on and on indefinitely (there is no smallest doll in the set). The dolls are constructed so that each one can stand on the head of the previous one. If I take out each doll and stand it on the head of the previous doll, what will be the height of the "matryoshka tower"?

- 6.2.** Construct a flow chart diagram for the phone number lookup process described in class. Be sure to put in a decision procedure to end the process.
- 6.3.** Apply the phone number lookup process to the faculty section of the University phone directory, using either Kerckhove or Elhai as the name to look up. Simplify your life by using some estimation of the midpoint name. Which name is found in the fewer number of steps? Among the faculty and staff, whose name(s) are the "worst" to look up (i.e. require the greatest number of steps)?

The process is called a "recursive binary search" and binary evokes the fact that with each successive function call we reduce the length of the input list by half.

- 6.4.** Provide a recursive set of instructions and/or a flow chart diagram by means of which Achilles may catch the Tortoise in their race from Dialog 1. To be recursive, you should be able to include a phrase like "repeat the instructions above, substituting the current state for the initial state" somewhere in your "recipe for success".
- 6.5.** Provide a recursive set of instructions and/or a flow chart diagram by means of which the Tortoise may continue to confound Achilles' attempts to reach agreement that "if (A) and (B), then (Z)" in Dialog 2. To be recursive, you should be able to include a phrase like "repeat the instructions above, substituting the current state for the initial state" somewhere in your "recipe for success".

- 6.6.** Provide a recursive set of instructions by means of which the full generating tree for the **MIU**-system (p 40) may be produced. Start with **MI** as the sole axiom... . To be recursive, you should be able to include a phrase like "repeat the instructions above, substituting the current state for the initial state" somewhere in your "recipe for success".
- 6.7.** Describe an everyday problem that has a recursive solution. Write out a recursive set of instructions that solves the problem. Post both problem and solution on the discussion board.
- 6.8.** Return to Exam 1, problem 10 (the **EXAM1**-system) and the posted response to that problem. See if you can show that the number of theorems at the  $n$ th level of the generating tree is equal to the  $n$ th Fibonacci number.
- 6.9.** Provide a set of rules to be used with the symbols of the **MIU**-system and the sole axiom **MU** (not **MI**) that can generate as many strings as possible that are NOT theorems in the **MIU**-system (with **MI** as the sole axiom). Post at least one rule to the Discussion Board.