

MATH 195: Gödel, Escher, and Bach (Spring 2001)

Problem Set 4: To be discussed Thursday, February 8

1. Consider each of the strings shown below. If the string is a theorem, then provide a derivation. Otherwise, state how you determined that it is not a theorem.

a. ---t---q----- b. --t---q----- c. ---tq

2. Find a way to list all theorems of the **tq**-system. Use that list to show that every theorem in the system has a corresponding true arithmetic statement in which two positive integers are multiplied. (Consider using the diagonal strategy, equivalent to the bucket strategy outlined by Hofstadter for the **pq**-system; alternatively consider how to produce the generating tree and what it would look like).
3. Devise a machine that uses only allowed typographical operations to test whether a well-formed string is an axiom of the **tq**-system.
4. How many axioms does the C-system have?
5. Can you characterize the set of well-formed strings in the C-system?
6. Devise a bottom-up approach using only allowed typographical operations to test whether the string **Cz** is a theorem (the test should work for any hyphen string **z**, **no matter how long**). Make sure that the procedure has a defined end point. By "bottom-up" I mean that you should start with one or more axioms and work from there. HINT: Decide first what axioms you will need for a string of hyphens of length **z**. Then decide what you must do with these axioms to ensure that you will derive **Cz**, if that is possible.

Illegally Characterizing Primes

7. Make a rule similar to the proposed rule on p.66 but using the elements of the MU puzzle. Show how your rule violates the restrictions of typographical operations.

Recursively Enumerable Sets vs. Recursive Sets

8. Define what it means to be a recursively enumerable set and provide an example. What's the difference between a recursively enumerable set and a recursive set?
9. How is the set of positive integers 1 3 7 12 18 26 ... given on p.73 similar to the FIGURE-FIGURE figure? Do your best to formulate (in plain English) a rule that would generate this set of positive integers.

Primes as Figure Rather than Ground

10. Obtain a derivation of **P - - -** using a Top-Down approach (that is, work backwards from **P - - -**) and the system for generating primes that is described on p73-74. HINT: Explain why you'd like to obtain the string **- - - DF - -**. FURTHER HINT: You won't need the third RULE.
11. What would the term "theorem schema" mean? Give as precise a definition as you can. In the **tq**-system, the string **--t x q xx** might qualify as a theorem schema. Explain.

12. Your company has purchased Unintendo, a now bankrupt manufacturer of electronic games. Along with the deal came the device used to make Game-Person, the once wildly successful toy. You realize that if you can give the toy a face-lift for the new savvy kid market, you could make a mint.

The device is ingenious, scanning the printed circuit board of an input Game-Person toy and using it (with the help of the human operator) to output a slightly different circuit board for a new toy. That way, no toy is exactly the same, and kids pester their parents to buy more and more copies. The only people from Unintendo who know how this device works have cashed in their stock options and are now lolling under the hot Caribbean sun. Their answering machine just gives you sounds of crashing waves.

So it's up to you and the woefully insufficient training guide they left you. The guide is nothing more than a test circuit board with 10 different series of components plus instructions as to what button on the machine you're supposed to push for each series. The circuit boards consist of transistors (**t**), capacitors (**c**), and a strange component you can't identify (**#**). You put the test circuit board into the machine and dutifully push the buttons the guide calls for. You see that you need to understand the meaning of the buttons and the rules that govern what kind of circuit board the device produces. The table below shows the results of your experiment.

Results of Experiments with Game-Person Device		
Input circuit	Button pushed	Output circuit
tc#tc	green	tc#tc#tc
#t	blue	#tc#
#c	blue	#ct#
cttct	red	none
cct#ttt	green	cct#ttt#ttt
ttt	red	none
t#t	green	t#t#t
tt#cccc	green	tt#cccc#cccc
#ccctt	blue	#cccttcttt#
cttc	red	none

- Describe the actions instigated by each of the three buttons.
- Describe the strategy or strategies that YOU used in order to solve this problem.
- You have determined that the presence of "**tct#**" in the circuitry changes the resulting toy so that it emits Z-rays, known to increase human's compulsive behavior. What circuit can you give to the device to output "**tct#**" and what button should you push?

Do you see that parts **a-c** mimic our attempts to understand the phenomena we encounter: we make observations, assume the data come from some underlying systematic rules, then try to describe the system. Be aware through the rest of the semester of the strategies you employ while dreaming up systems that are consistent with observed data.