```
wonder[x_Integer] := If[Mod[x, 2] == 0,
   (*then*) x / 2,
   (*otherwise *) 3 x + 1];
```

Recall that x is "wondrous" if the sequence obtained by following this algorithm produces 1 eventually. Let's call the minimum number of steps required to produce 1 from x the "wonder number of x"

```
NestList[wonder, 1, 3]
```

```
Out[90]= {1, 4, 2, 1}
```

Theorem: If x is a power of 2, then x is wondrous.

```
NestList[wonder, 2, 1]
```

```
Out[91]= {2, 1}
```

Theorem: The WonderNumber of 2^N is N.

Corollary: Every number is the wonder number of some other number.

```
NestList[wonder, 3, 7]
```

```
Out[92]= {3, 10, 5, 16, 8, 4, 2, 1}
```

```
NestList[wonder, 5, 5]
```

```
Out[94]= {5, 16, 8, 4, 2, 1}
```

```
NestList[wonder, 6, 8]
```

```
Out[95]= {6, 3, 10, 5, 16, 8, 4, 2, 1}
```

Theorem: If x is even and $x/2$ is wondrous, then x is wondrous.
Moreover, WonderNumber(x) = WonderNumber($x/2$) + 1.

This means that if we proceed in sequence from here on, only odd numbers need be considered.

```
NestList[wonder, 7, 16]
```

```
Out[97]= {7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1}
```

```
NestList[wonder, 9, 19]
```

```
Out[99]= {9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1}
```

The wondrous numbers we have so far are
{1,2,3,4,5,6,7,8,9,10,11,13,14,16,17,20,22,26,28,34,40}
with corresponding wonder numbers
{3,1,7,2,5,8,16,3,19,6,14,10,17,4,12,7,15,10,18,13,11}

The next thing worth noting seems to be the following.

```
In[107]:= NestList[wonder, 18, 20]

Out[107]= {18, 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1}

In[106]:= NestList[wonder, 19, 20]

Out[106]= {19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1}
```

Theorem: 18 and 19 are the first consecutive wondrous integers which have the same wonder number.

Not much interesting happens until $x = 27$.

```
In[127]:= NestList[wonder, 27, 111]

Out[127]= {27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364,
           182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263,
           790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377,
           1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238,
           1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077,
           9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244,
           122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1}
```

The wonder number of 27 is 111. The largest wondrous number so far discovered by me today is 9232.

wonder's