

## MATH 195: Gödel, Escher, and Bach (Spring 2001)

Notes and Study Questions for Thursday, April 5

Reading: *Chapter IX: Mumon and Gödel* (pp.269-272):

The reading takes us from the Gödel numbering to the Gödel statement “This string,  $G$ , is not a theorem of TNT”. Please refer now to the diagram at the top of page 271. The diagram is called the *Central Dogma of Mathematical Logic*.

The trick described in the following section of text is to find a string  $G$  of TNT which goes (by the arrow on the left) to an interpretation in Number Theory (recall that Hofstadter symbolizes number theory by  $N$ ). This interpretation of  $G$  in Number Theory is to be uncovered as an actual true fact. The string  $G$  is to have the additional property that its number-theoretic counterpart in  $N$  subsequently goes (by the arrow on the right) to a statement of **Meta-TNT** whose interpreted meaning as a statement *about* TNT is the statement “The string  $G$  is not a theorem of TNT”.

Showing that there exists such a string,  $G$ , within TNT is postponed until Chapters XIII and XIV. Here, Hofstadter discusses the implications of  $G$ 's existence. I need to refer to that diagram on the top of page 271 frequently in order to remind myself where I am.

**TNT-Numbers through End of Chapter (pp.269-272):**

**Study Questions** (Be prepared to discuss them **April 5**)

Most of these have to do with being able to tell what's what; an important skill.

SQ1. True or False: 30 is a number.

SQ2. True or False: 30 is a TNT-number.

SQ3. True or False: Every TNT-number is a number.

SQ4. True or False: If  $x$  is a TNT-number, then  $x$  is the Gödel Number for some particular string of TNT.

SQ5. True or False: If  $x$  is the Gödel Number for some particular string of TNT, then  $x$  is a TNT-number.

SQ6. Find a number, other than numbers in the book, that is a TNT-number.

SQ7. Find a number, other than numbers in the book, that is presumably not a TNT-number.

SQ8. Hofstadter uses **N** to symbolize

- a) a particular natural number
- b) the set of facts of number theory
- c) the set of well-formed strings within **TNT**
- d) the set of statements about **TNT**

SQ9. Why is it more appropriate to refer to a “fact” of number theory than to a “theorem” of number theory?

SQ10. The diagram of the *Central Dogma of Mathematical Logic* is very important. In this diagram, “reality” is played by

- a) **TNT**
- b) **N**
- c) meta-**TNT**
- d) None of the above

SQ11. The diagram of the *Central Dogma of Mathematical Logic* is very important. In this diagram, “formal system” is played by

- a) **TNT**
- b) **N**
- c) meta-**TNT**
- d) All of the above

SQ12. Indicate which part of the diagram corresponds to the statement

$361,123,666,112,123,666,323,111,123,123,666$  is a **TNT**-number

- a) **TNT**  $\Rightarrow$  **N**
- b) **N**  $\Rightarrow$  meta-**TNT**
- c) **TNT**  $\Rightarrow$  **N**  $\Rightarrow$  meta-**TNT**
- d) None of the above

SQ13. Indicate which part of the diagram corresponds to the statement

$(S0 + S0) = SSO$  is a theorem of **TNT**

- a) **TNT**  $\Rightarrow$  **N**
- b) **N**  $\Rightarrow$  meta-**TNT**
- c) **TNT**  $\Rightarrow$  **N**  $\Rightarrow$  meta-**TNT**
- d) None of the above

Two more questions that will take some thought.

The sentence “Now it occurs to us that this new number theoretical predicate (  $a$  is a TNT-number ) is *expressible* by some string of TNT with one free variable.” is crucial. These questions might help you figure out what the sentence is saying.

SQ14. The statement  $a$  is a TNT-number is best described as

- a) a problem about  $N$
- b) a fact of number theory
- c) a theorem of TNT
- d) a Gödel Number

SQ15. Which diagrammatic representation below is the genesis of this crucial idea? (see SQ14).

- a)  
    { the set of problems about  $N$  }  $\rightarrow$  { the set of facts of number theory }  
  via Gödel Numbering
- b)  
    { the set of facts of number theory }  $\rightarrow$  { set of well-formed formulae of TNT }  
  via interpretation of symbols
- c)  
    { the set of theorems of TNT }  $\rightarrow$  { set of well-formed formulae of TNT }  
  as a subset
- d)  
    { the set of theorems of TNT }  $\rightarrow$  { the set of facts of number theory }  
  via interpretation of symbols  
  and incorporation of reasoning modes in TNT

SQ16. Has the Dogma Buddha-nature?