

MATH 195: Gödel, Escher, and Bach (Spring 2000)

Notes and Study Questions for Thursday, March 1

Reading: pp.188-190 (or more accurately, same as Tuesday's plus a tad more)
Truth Tables (in Additional Material)

We've bit off quite a bit for the moment. Let's digest what we have. You should have the following goals for Thursday:

1. Understand the symbols of the Propositional Calculus (PC) and their literal interpretations
2. Understand how to construct a well-formed string within PC
3. Understand how to recognize a well-formed string within PC
4. Be able to translate some English sentences into PC
5. Be able to make up English sentences that correspond to statements within PC
6. Be able to use the Fantasy Rule to derive simple statements within PC
7. Feel comfortable about the logical reasonableness of the Rules of Inference

This is a lot. To help you organize your tasks, I've provided a summary of PC at the end of these notes.

Understand how to construct and recognize a well-formed string within PC

1. Do Problem Set 7, #1 (note that the book provides the answers).
2. Do Problem Set 7, #2

Be able to translate some English sentences into PC and vice versa

3. Do Problem Set 7, #4a, 4b, 4e, 4f (what does "but" mean?), 4g, 4I, and 4l (what does "only when" mean?)
4. Translate one of the strings (your choice) from Problem Set 7, #5.
5. Go through each of the Rules of Induction and make up corresponding English sentences

Be able to use the Fantasy Rule to derive simple statements

6. Do Problem Set 7, #7
7. Go through the derivation on p.189. For now simply verify that each line makes sense in isolation (i.e. uses an applicable rule). Don't try to understand how the proof works as a whole.

Truth tables

Our purpose in studying Propositional Calculus is to convince ourselves that by restricting the language to AND, OR, NOT, and IF ... THEN and by insisting on very specific interpretations of those terms in terms of truth and falsehood, we could mechanize at least the rudiments of our thought process. Hofstadter is careful to focus on the mechanics, hardly mentioning truth in this chapter. For us, it will be a little easier, and a lot more applicable once the course ends, to incorporate truth and falsehood by means of agreed upon TRUTH TABLES right from the beginning of our discussion.

Truth tables are relatively straightforward tools to evaluate a proposed theorem. Unlike derivations, they don't require much in the way of inspiration, though doing them can be tedious with complex statements. The page in Additional Material on truth tables should get you going on the topic, and we'll also discuss them in class.

The Propositional Calculus (Summary)

Symbols

Atom particles:	P Q R	
Atom suffix:	'	(prime)
Prefix operation:	~	(not)
Infix operations:	^ v É	(and, or, if-then/implies)
Infix op boundaries:	< >	(begin op, end op)
Fantasy boundaries:	[]	(push, pop)

Rules of Formation

RULE #0: All atoms are well-formed	atoms
RULE #1: If x is well-formed, then so is ~ x	~ x
RULE #2: If x and y are well-formed, then so is < x ^ y >	< x ^ y >
RULE #3: If x and y are well-formed, then so is < x v y >	< x v y >
RULE #4: If x and y are well-formed, then so is < x É y >	< x É y >

Axioms

None

Rules of Inference

Rule	Prior Theorem	Consequent Theorem
JOINING RULE	x and y	< x ^ y >
SEPARATION RULE	< x ^ y >	x and y
DETACHMENT RULE	x and < x É y >	y
CONTRAPOSITIVE RULE	< x É y > <~ y É~ x >	<~ y É~ x > < x É y >
DE MORGAN'S RULE	<~ x ^~ y > <~ x v y >	~< x v y > <~ x ^~ y >
SWITCHEROO RULE	< x v y > <~ x É y >	<~ x É y > < x v y >
DOUBLE-TILDE RULE ^a	Any string with ~~ Any string	Same string with one less ~~ Same string with one more ~~
FANTASY RULE	If y can be derived given x	< x É y >

^aValid only if resulting string is well-formed