

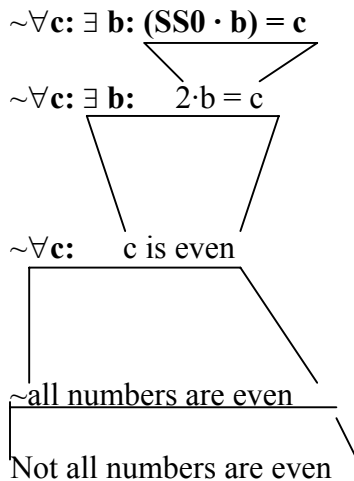
MATH 195: Gödel, Escher, and Bach (Spring 2001)

Typographical Number Theory: Translations and Well-Formed Formula

Hints for translating TNT sentences to English

- Don't try to understand an entire sentence all at once
- Understand chunks of a sentence; replace chunks as you go along
- Usually it's easier to go right to left
- Test reasonableness as you go along

Example



*Two times $b = c$... reasonable, but not so interesting.
 There **EXISTS** some b for which this is true...
 If so, then c must be even*

*For every c , c is even...doesn't sound reasonable to me,
but that's what it says. If it's true for every number c ,
 then it's true for all numbers.*

*...which is a true statement. Note that negating the
 statement, "All numbers are even" does **NOT** give
 "No numbers are even."*

Well-Formed Formula in TNT

Do the symbol strings below qualify as "well-formed formulas of TNT"?

- 1) $\exists b: \langle a = SS0 \wedge (a + b) = S0 \rangle$ compare with $\langle a = SS0 \wedge \exists b: (a + b) = S0 \rangle$
- 2) $\forall a: \langle a = SS0 \supset \exists b \rangle$
- 3) $(\exists b: \exists c: SSb \cdot SSc = \sim a)$ compare with $(\sim \exists b: \exists c: SSb \cdot SSc = a)$
- 4) $\langle S0 = 0 \vee S0 \rangle$