List-coloring the Square of a Subcubic Graph

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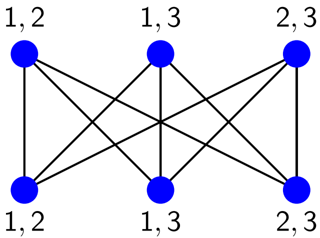

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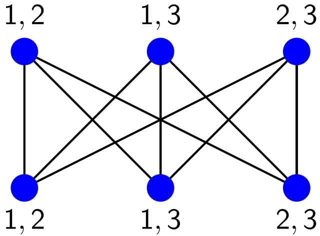

G^2 (square of G): formed from G by adding edges between vertices at distance 2.

Conjecture [Wegner 1977]

If G is planar and $\Delta(G) = 3$, then $\chi(G^2) \leq 7$.

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$\Longrightarrow$ $\chi_l(G^2) \leq 7$ if G is planar and $\Delta(G) = 3$.

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Plan

- get an upper bound on $\overline{d}(G)$
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- list forbidden subgraphs
- use discharging to show that if G does not contain any forbidden subgraph, then the bound on $\overline{d}(G)$ does not hold

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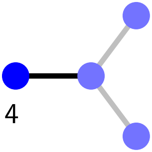

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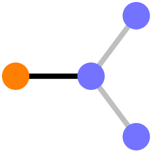

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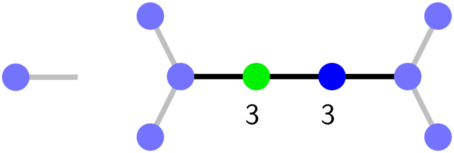

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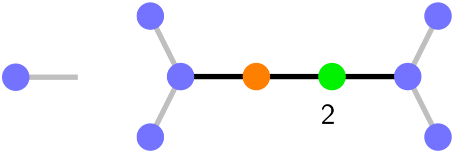

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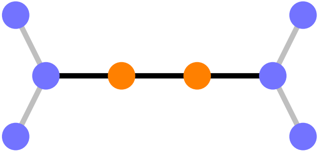

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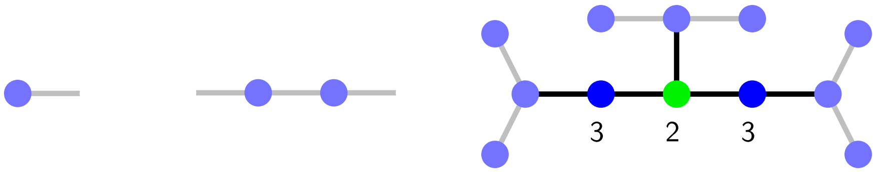

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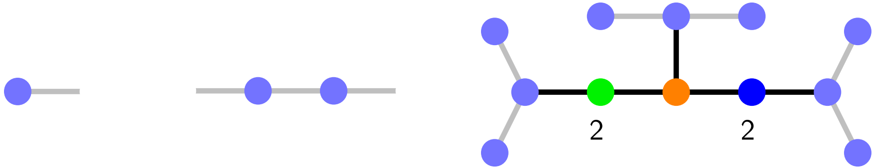

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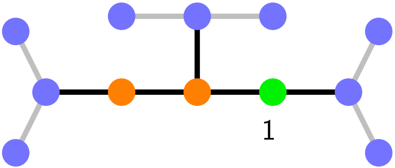

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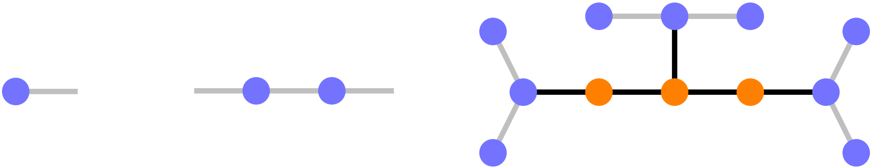

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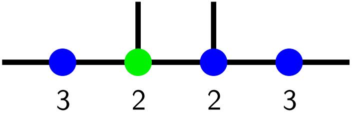

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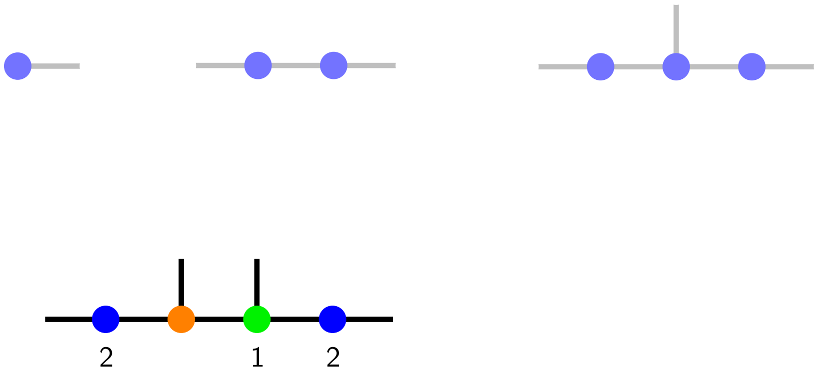

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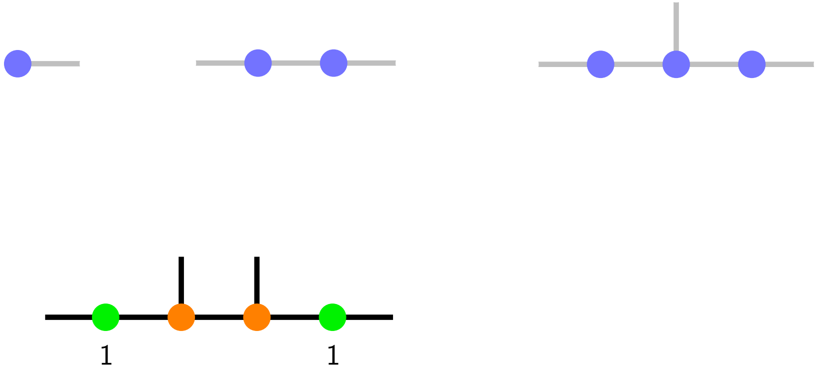

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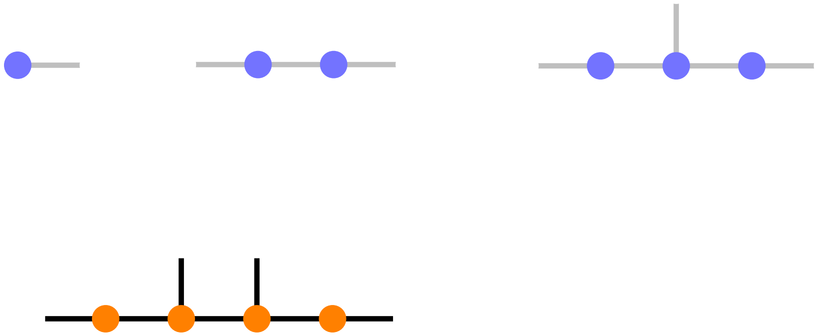

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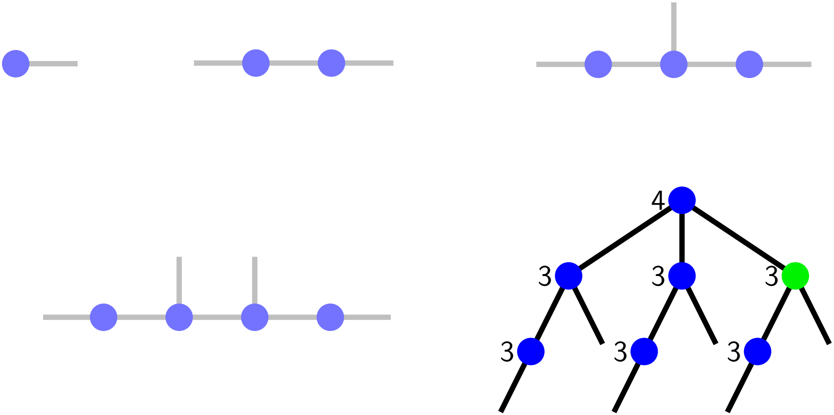

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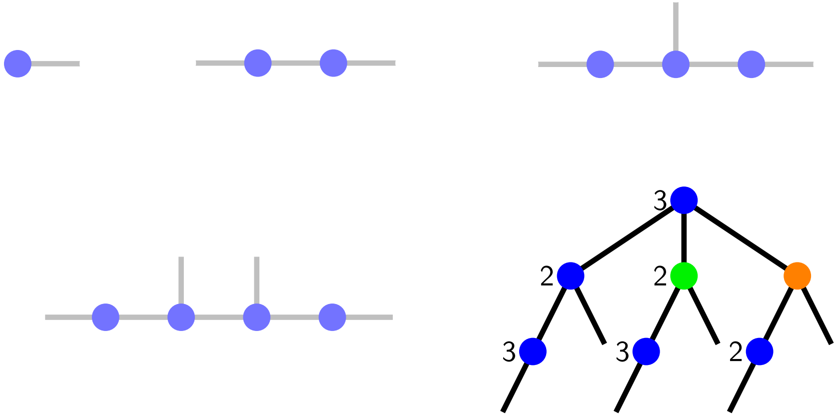

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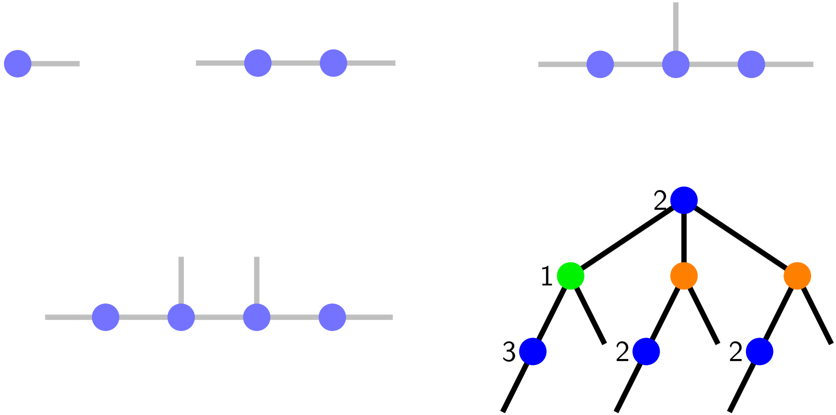

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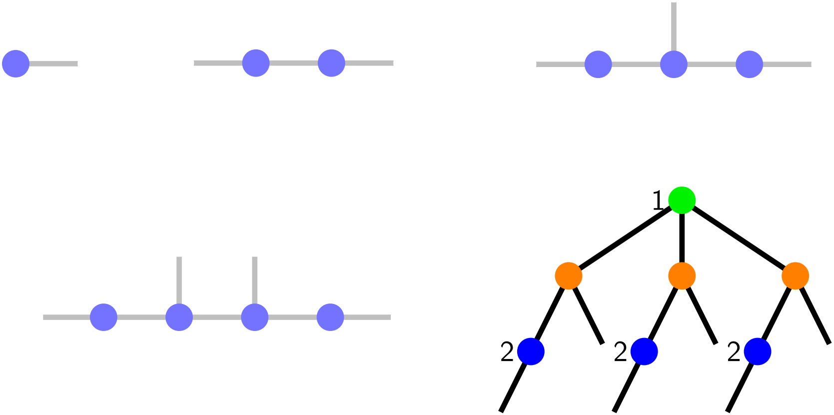

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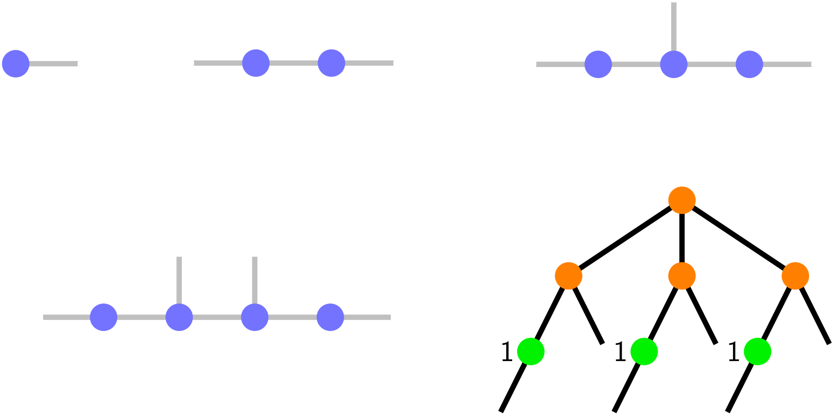

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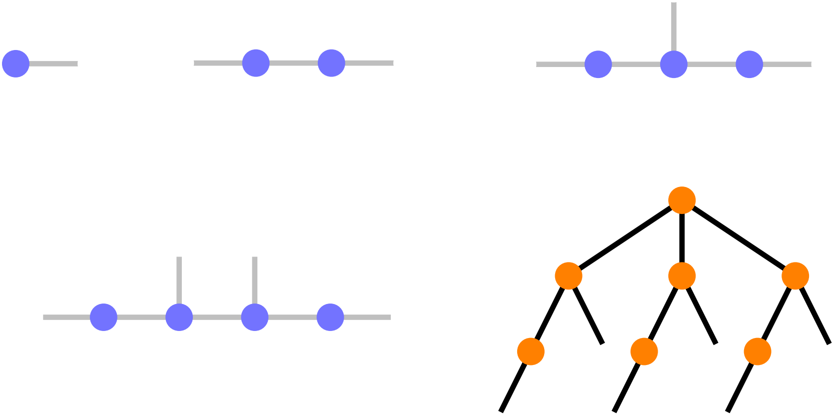

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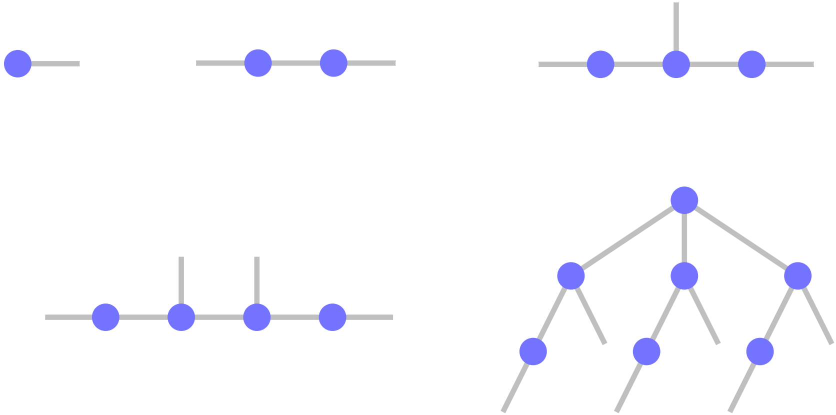

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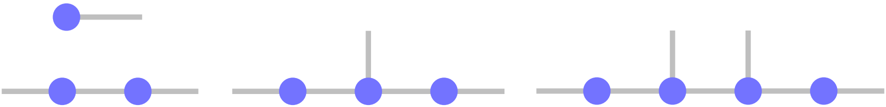
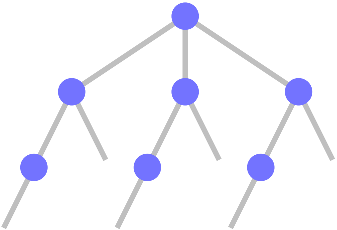


Corollary 6:

Let $M_1(v)$ and $M_2(v)$ denote the number of 2-vertices at distance 1 and 2 from v.



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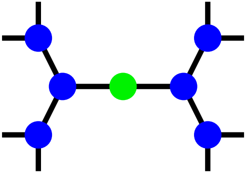

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Show that $\mu^*(v) \geq 2\frac{4}{5}$ for each vertex v.

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If G is planar, $\Delta(G) = 3$, girth ≥ 7, then $\chi_l(G^2) \leq 7$.

Corollary 6:

If v is a 2-vertex, then $M_1(v) = M_2(v) = 0$.

3-vertex, then $2M_1(v) + M_2(v) \leq 2$.

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We use a discharging argument with $\mu(v) = d(v)$.

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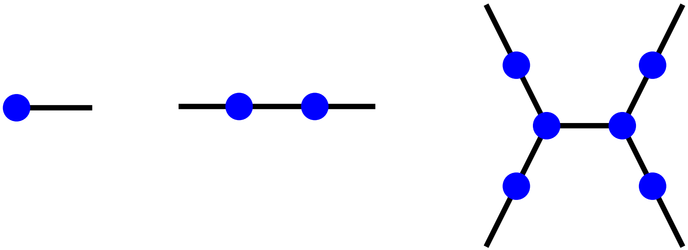

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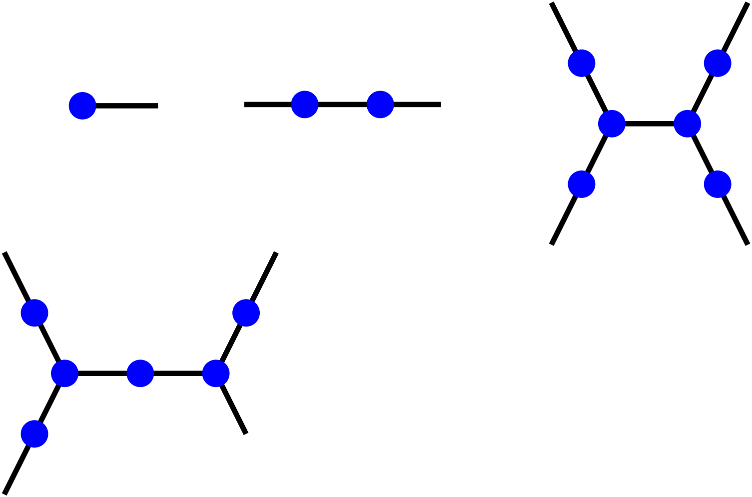

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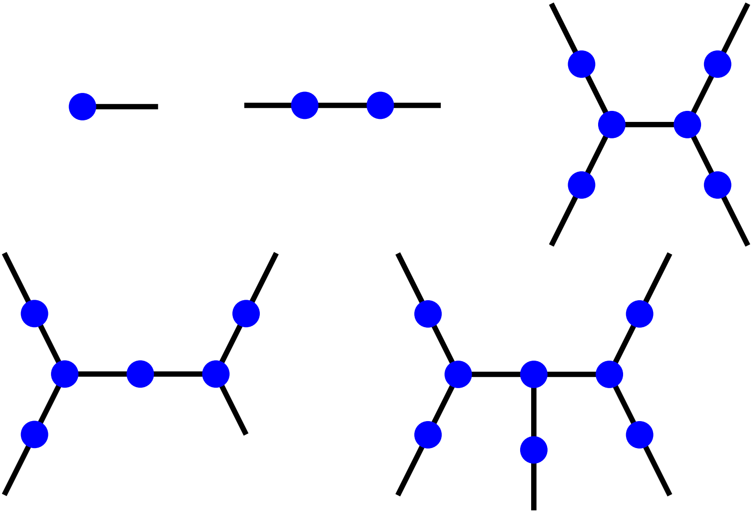

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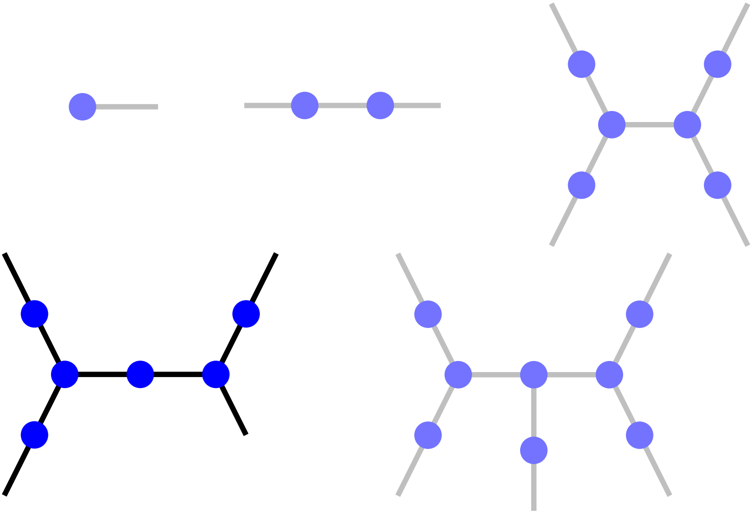

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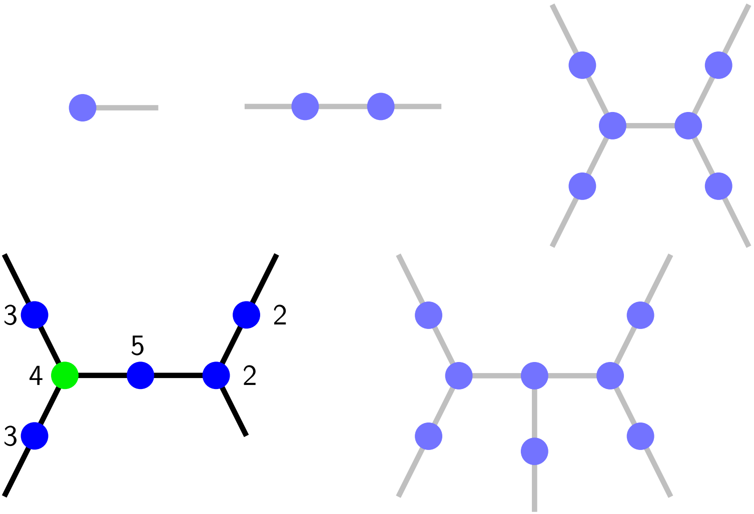

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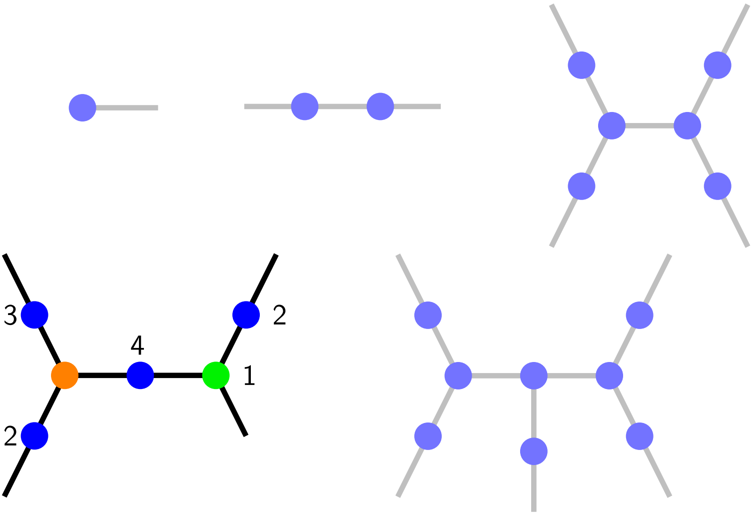

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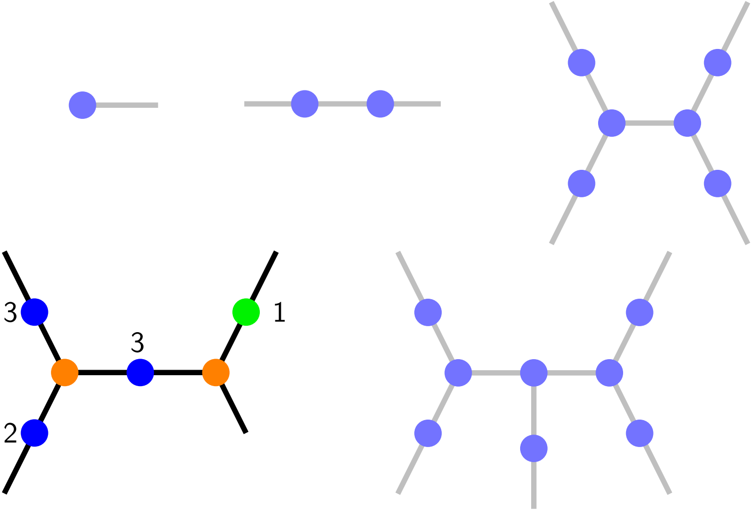

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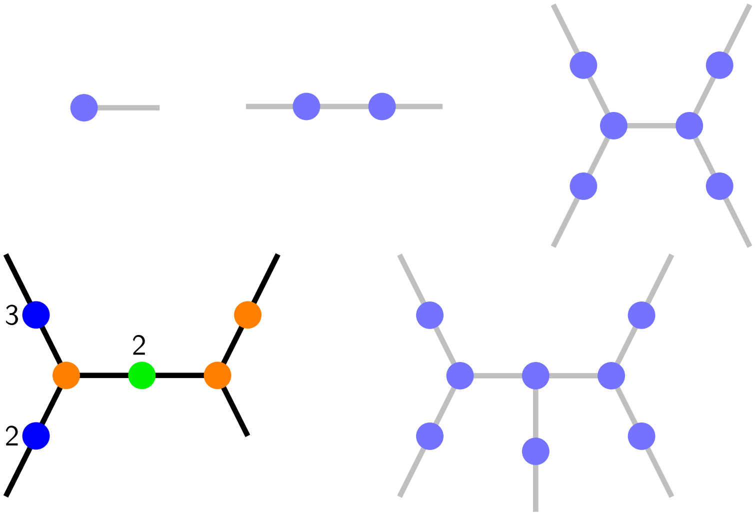

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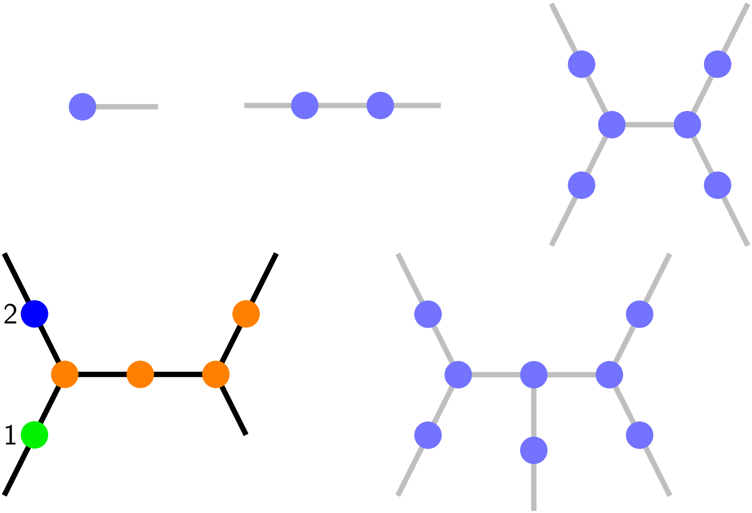

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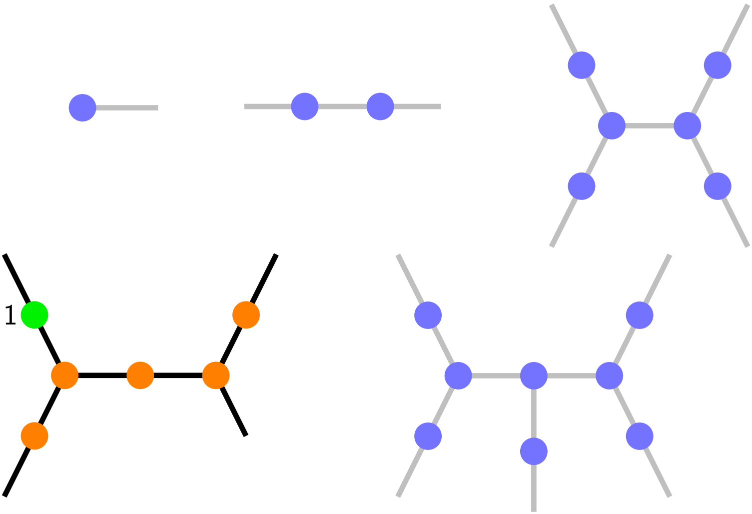

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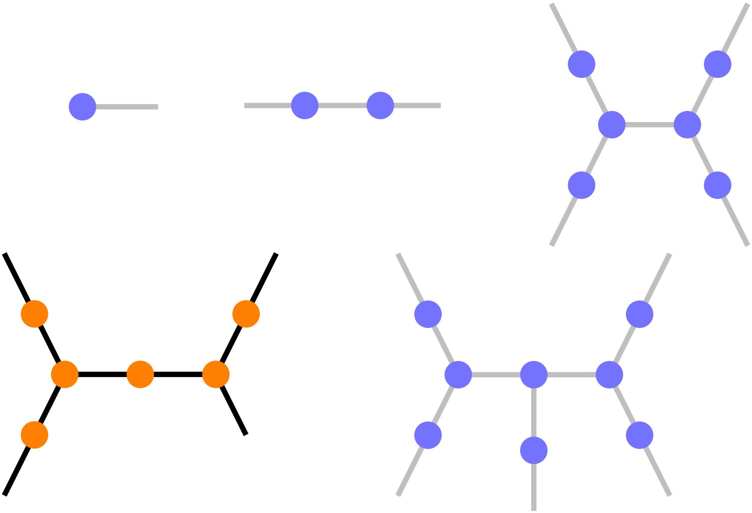

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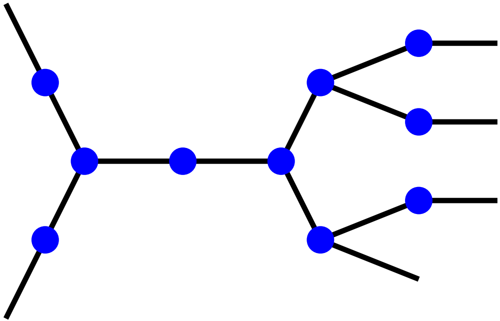

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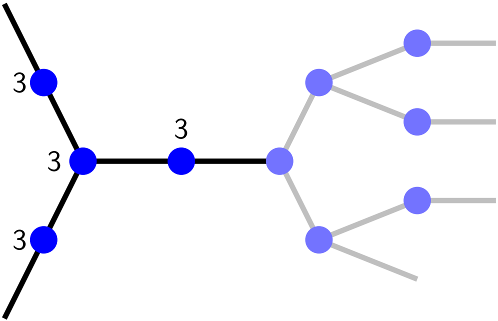

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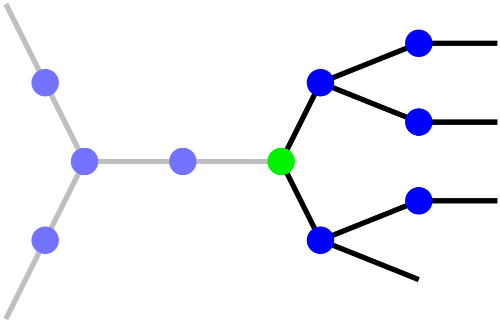

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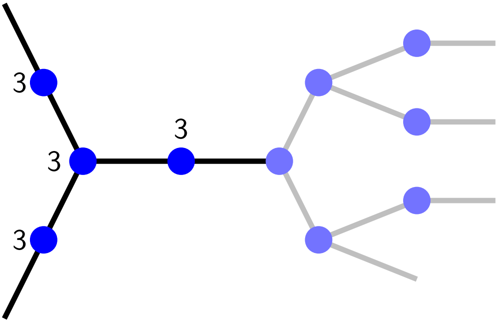

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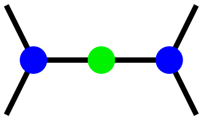

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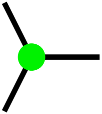

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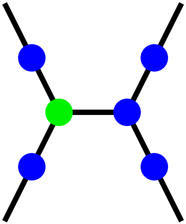

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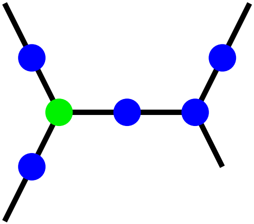

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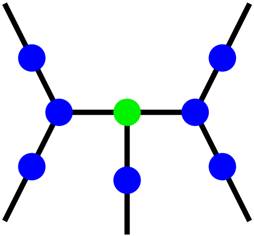

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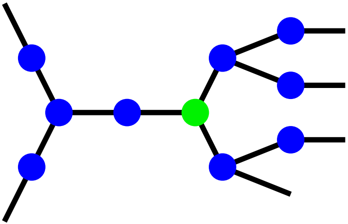

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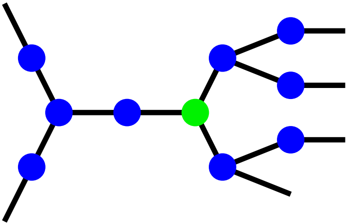

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3. Is it true that every graph G satisfies $\chi_l(G^2) = \chi(G^2)$?

Thank you!

Any Questions?