Spies and Revolutionaries

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Setup: $r$ revolutionaries play against $s$ spies on a graph $G$. Each rev. moves to a vertex, then each spy moves to a vertex.
Introduction

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![Diagram of a graph with revolutionaries (red) and spies (green) with labels showing the setup and observations.](image-url)
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So we assume \( \lfloor r/m \rfloor \leq s < |V(G)| \).
Spy-friendly Graphs

**Def:** A graph $G$ is spy-friendly if the spies win on $G$ for all integers $m, r, s$ such that $\lfloor r/m \rfloor \leq s$. 

**Thm 1:** All paths are spy-friendly.

**Pf:** One spy follows each $m$th rev. When rev's move, spies repeat.

**Ex:** $P_9$ is spy-friendly. Consider $m = 3, r = 13, s = 4$.

**Ex:** $C_5$ is not spy-friendly. Consider $m = 2, r = 3, s = 1$.

**Lemma:** For $k \geq 4$, $C_k$ is not spy-friendly. . . but it's very close.
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![Diagram of a path graph with vertices connected by edges]
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```
  r r r  r r r  r  r r r  r  r r r  r
  ⬛ ⬛ ⬛  ⬛ ⬛ ⬛  ⬛  ⬛ ⬛ ⬛  ⬛ ⬛ ⬛
```
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![Diagram of a path graph with nodes labeled and arrows indicating movement. The diagram shows a sequence of moves where spies follow the $m$th rev.]
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\[ \begin{array}{cccccccc}
  r & r & r & r & r & r & r & r \\
 s & s & s & s & s & s & s & s \\
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![Diagram of a path graph $P_9$ with nodes labeled and arrows indicating the movement of spies. The spies win for the given values of $m$, $r$, and $s$.](image-url)
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[Diagram of a path graph with nodes and edges labeled to illustrate the spy-friendly property]
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![Graph Diagram]
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**Thm 3:** All cycles are nearly spy-friendly.

**Pf. idea:** Same as for paths; one spy follows each $m$th rev.

**Ex:** Consider $C_8$, when $m = 2$, $r = 8$, and $s = 4$. 

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Playing on the Sun

**Def:** A sun is a cycle with paths hanging off some vertices.
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**Thm 4:** All suns are nearly spy-friendly. ($\lceil r/m \rceil$ spies can win)

**Pf. idea:** Use cycle strategy on cycle and path strategy on paths.

**Ques:** What if rev's move back and forth from the cycle to paths?
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**Def:** A *sun* is a cycle with paths hanging off some vertices.

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**Key Insights (for cycles):**
Say the spies have a good position on a cycle.
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- If $m$ new rev’s and 1 new spy appear on the same vertex of the cycle, the new position is still good for the spies.
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- Same is true if \(m\) rev’s and 1 spy disappear.
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- Same is true if \(m\) rev’s and 1 spy disappear.
Playing on the Sun (cont’d)

Key Insights (for paths):
Suppose the spies have a good position on a path.

- If 1 new rev appears on far end, spies are still good unless \( m \mid r \).
- If \( m \mid r \), then spies are good if a new spy appears on the far end along with the new rev.
- We can also reverse these moves, and the spies remain good.
Key Insights (for paths):
Suppose the spies have a good position on a path.

Ex: $P_9$ with $m = 3$, $r = 13$, $s = 4$. 

![Diagram showing a path with dots representing positions for the spies and moves on the path.

- If 1 new rev appears on far end, spies are still good unless $m | r$.
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Key Insights (for paths):
Suppose the spies have a good position on a path.

Ex: $P_9$ with $m = 3$, $r = 14$, $s = 4$.

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Playing on the Sun (cont’d)

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Putting It All Together

**Thm 4:** All suns are nearly spy-friendly.

\[ \left\lceil \frac{r}{m} \right\rceil \text{ spies can win} \]

\[ \text{Pf. idea: Use cycle strategy on cycle and path strategy on paths.} \]

\( m = 2, r = 8, s = 4 \)
Putting It All Together

**Thm 4:** All suns are nearly spy-friendly. ([⌈r/m⌉ spies can win])
Putting It All Together

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Putting It All Together

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\[
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(m &= 2, \ r = 8, \ s = 4)
\end{align*}
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Putting It All Together

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**Putting It All Together**

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Putting It All Together

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