Planar graphs are 9/2-colorable and have big independent sets

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Joint with Landon Rabern
Slides available on my webpage

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George Washington
6 February 2015

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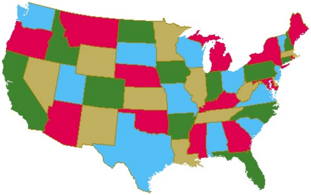

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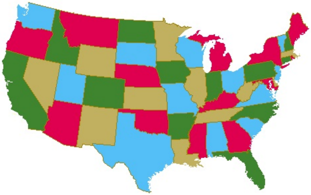

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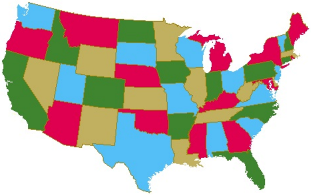

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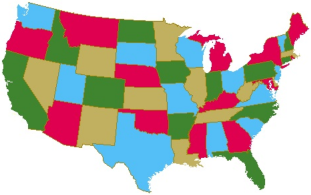

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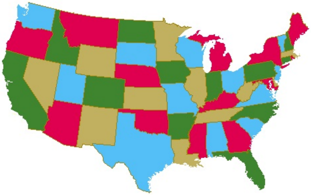

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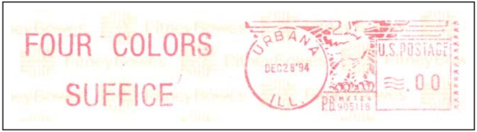

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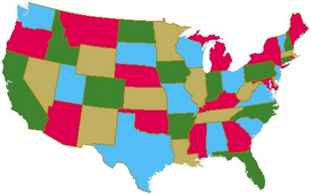

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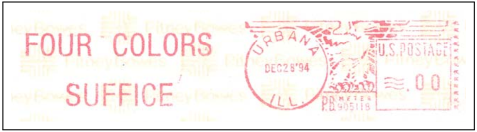

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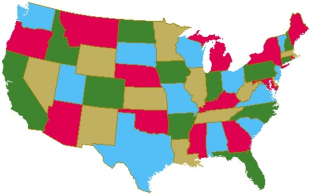

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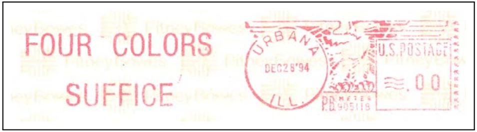

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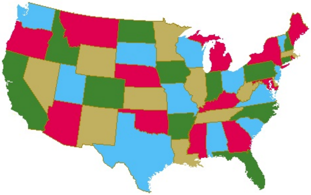

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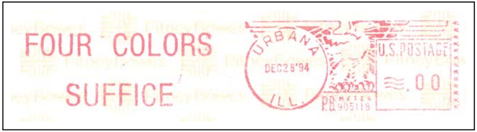

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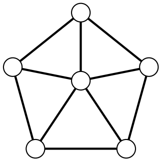

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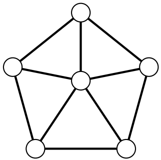

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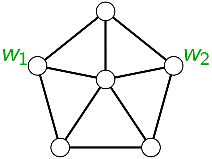

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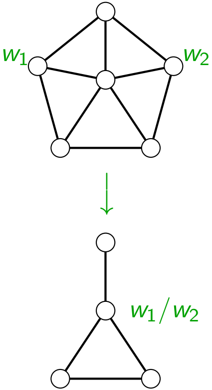

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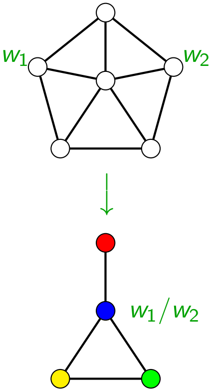

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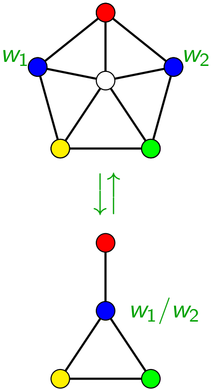

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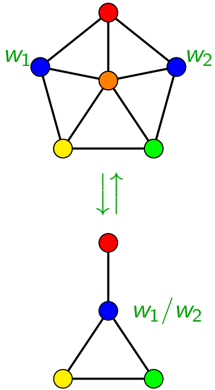

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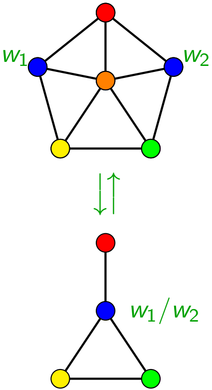

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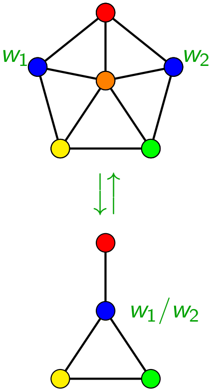

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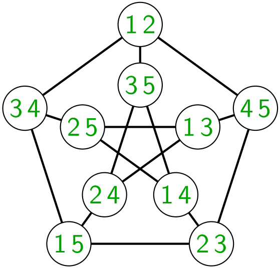

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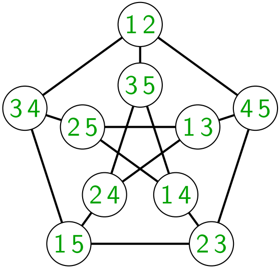

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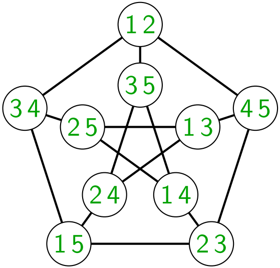

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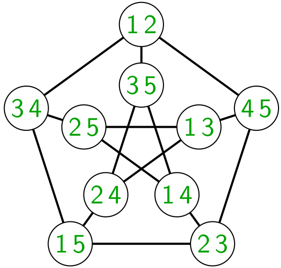

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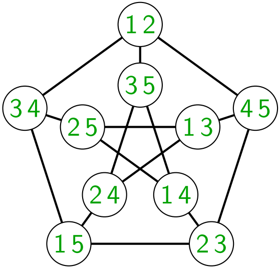

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Generalizes "coloring" to "coloring with graphs".

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   if so, then contract some non-adjacent pairs of neighbors;
   color smaller graph by induction, then extend to G

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2. has no separating triangle
3. can't have "too many 6^--vertices near each other"
   (13 cases, grouped into 3 lemmas by degree of central vertex)
   if so, then contract some non-adjacent pairs of neighbors;
   color smaller graph by induction, then extend to G

Finally, use discharging method (counting argument)
to show that every planar graph fails (1), (2), or (3).

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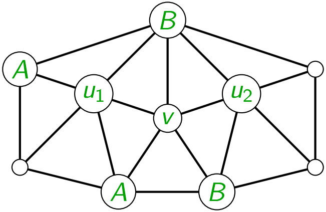

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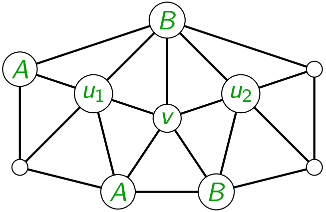

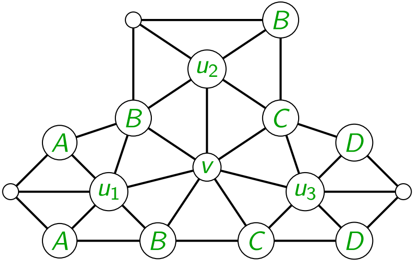

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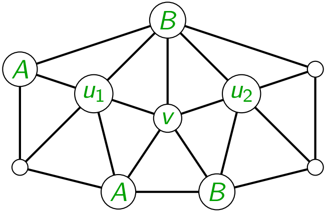

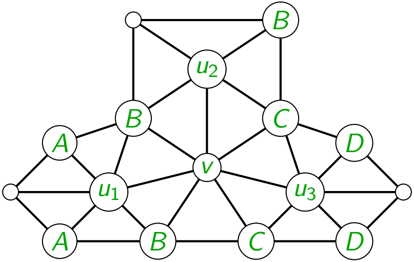

Rem: Reducible configuration proofs use only this Key Fact.

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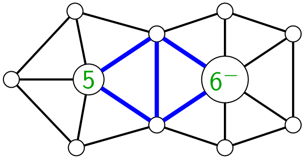

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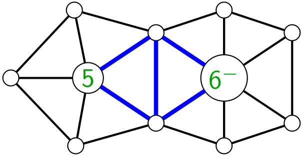

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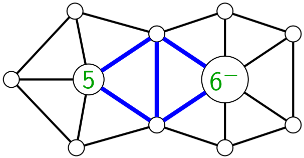

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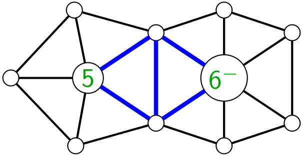

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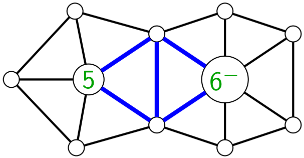

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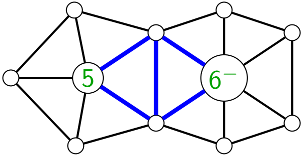

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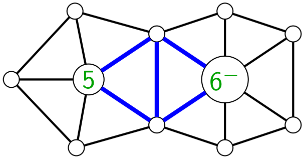

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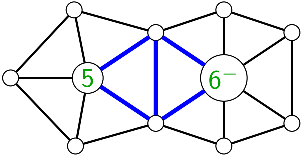

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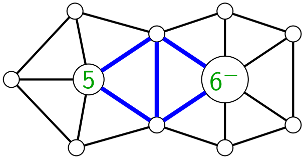

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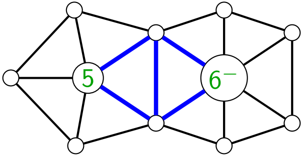

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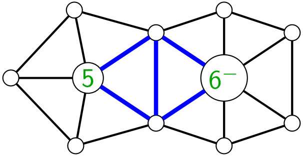

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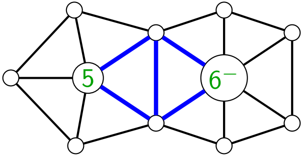

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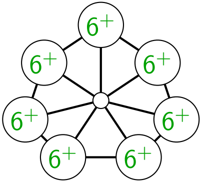

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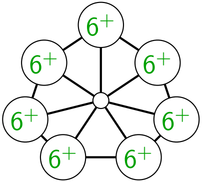

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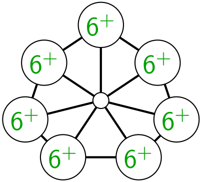

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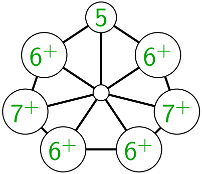

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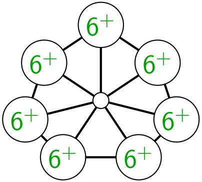

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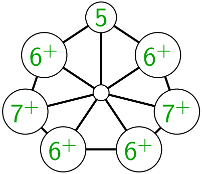

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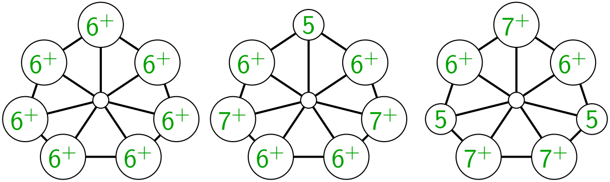

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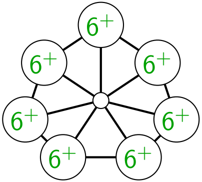

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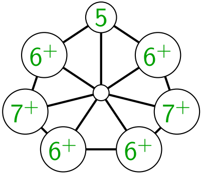

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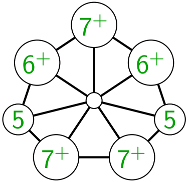

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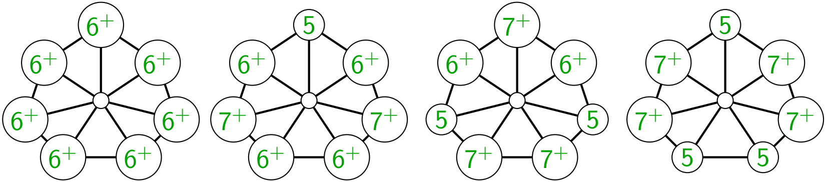
$d(v) = 8$**:** $8 - 6 - 4(\frac{1}{3}) - 4(\frac{1}{10}) > 0$.

$d(v) = 7$**:** Can give charge $7 - 6 = 1$.



$7(\frac{1}{10})$ $\quad$ $\frac{1}{3} + 4(\frac{1}{10})$ $\quad$ $2(\frac{1}{3}) + 2(\frac{1}{10})$

All Vertices are Happy

Discharging Rules

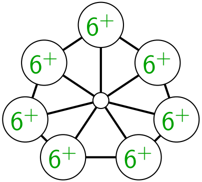
(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

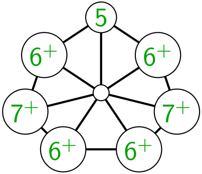
$d(v) \geq 9$**:** $d(v) - 6 - d(v)(\frac{1}{3}) = \frac{2}{3}(d(v) - 9) \geq 0$.

$d(v) = 8$**:** $8 - 6 - 4(\frac{1}{3}) - 4(\frac{1}{10}) > 0$.

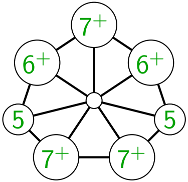
$d(v) = 7$**:** Can give charge $7 - 6 = 1$.



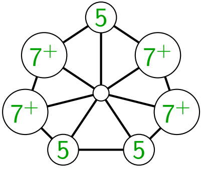
$7(\frac{1}{10})$



$\frac{1}{3} + 4(\frac{1}{10})$



$2(\frac{1}{3}) + 2(\frac{1}{10})$



$3(\frac{1}{3})$

All Vertices are Happy (continued)

All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:**

All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:**
Can give
net charge
$6 - 6 = 0$.

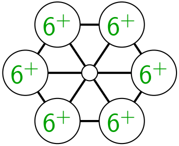
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:**
Can give net charge $6 - 6 = 0$.



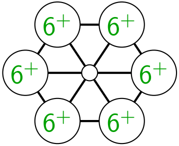
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$: Can give net charge $6 - 6 = 0$.



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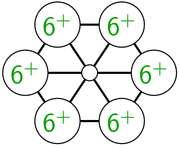
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:**
Can give net charge $6 - 6 = 0$.



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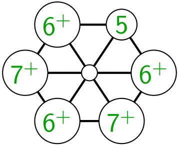


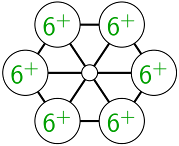
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:** Can give net charge $6 - 6 = 0$.



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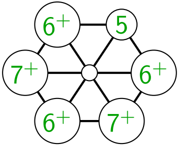


$\frac{1}{5} - 2(\frac{1}{10})$

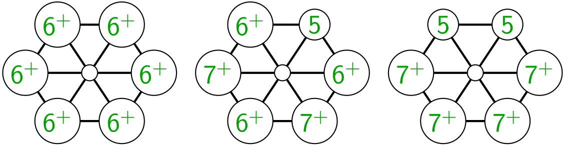
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:** Can give net charge $6 - 6 = 0$.



0 $\qquad$ $\frac{1}{5} - 2(\frac{1}{10})$

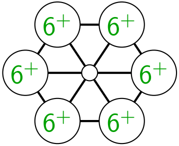
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$: Can give net charge $6 - 6 = 0$.



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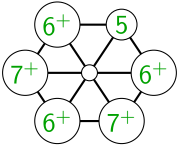
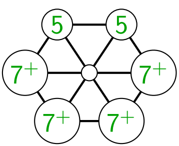


$\frac{1}{5} - 2(\frac{1}{10})$



$2(\frac{1}{5}) - 4(\frac{1}{10})$

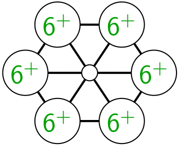
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

d(*v*) = 6: Can give net charge $6 - 6 = 0$.



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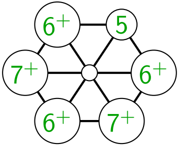
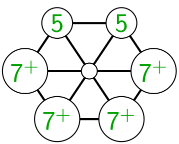


$\frac{1}{5} - 2(\frac{1}{10})$



$2(\frac{1}{5}) - 4(\frac{1}{10})$

d(*v*) = 5:

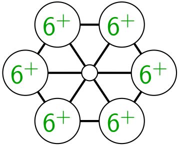
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:** Can give net charge $6 - 6 = 0$.



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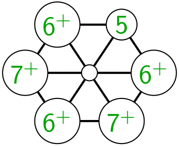
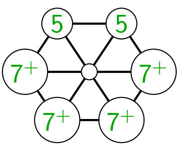


$\frac{1}{5} - 2(\frac{1}{10})$



$2(\frac{1}{5}) - 4(\frac{1}{10})$

$d(v) = 5$**:** Must get net charge $6 - 5 = 1$.

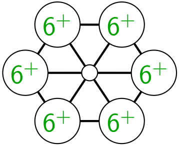
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$: Can give net charge $6 - 6 = 0$.



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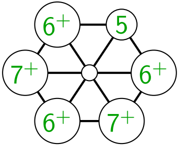
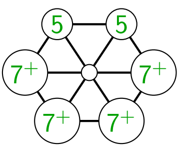
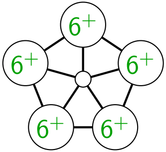


$\frac{1}{5} - 2(\frac{1}{10})$



$2(\frac{1}{5}) - 4(\frac{1}{10})$

$d(v) = 5$: Must get net charge $6 - 5 = 1$.



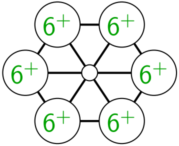
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:** Can give net charge $6 - 6 = 0$.



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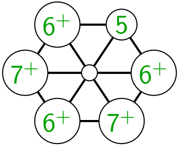
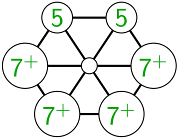
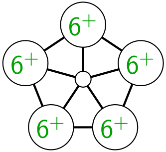


$\frac{1}{5} - 2(\frac{1}{10})$



$2(\frac{1}{5}) - 4(\frac{1}{10})$

$d(v) = 5$**:** Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$

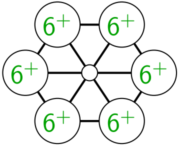
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$:
Can give net charge $6 - 6 = 0$.



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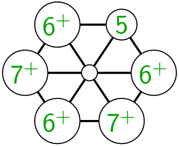
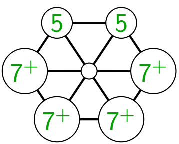
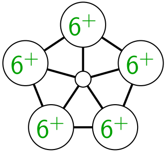


$\frac{1}{5} - 2(\frac{1}{10})$

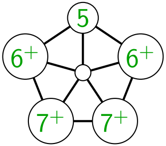


$2(\frac{1}{5}) - 4(\frac{1}{10})$

$d(v) = 5$:
Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$



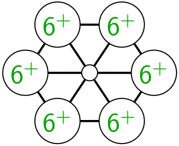
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$:
Can give net charge $6 - 6 = 0$.



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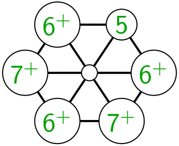
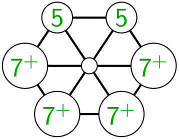
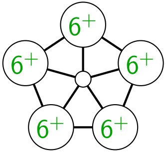


$\frac{1}{5} - 2(\frac{1}{10})$

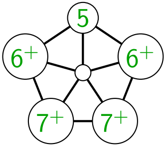


$2(\frac{1}{5}) - 4(\frac{1}{10})$

$d(v) = 5$:
Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$



$2(\frac{1}{3}) + 2(\frac{1}{5})$

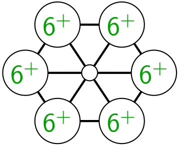
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$**:**
Can give net charge $6 - 6 = 0$.



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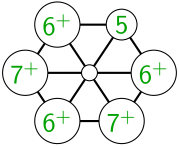
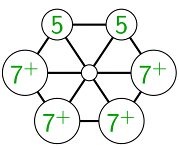
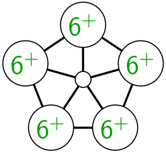


$\frac{1}{5} - 2(\frac{1}{10})$

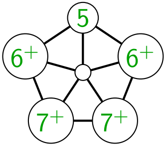


$2(\frac{1}{5}) - 4(\frac{1}{10})$

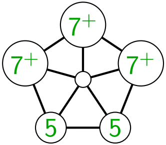
$d(v) = 5$**:**
Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$



$2(\frac{1}{3}) + 2(\frac{1}{5})$



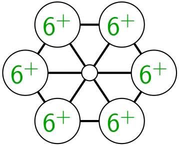
All Vertices are Happy (continued)

Discharging Rules

(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

$d(v) = 6$: Can give net charge $6 - 6 = 0$.



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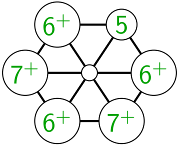
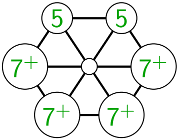
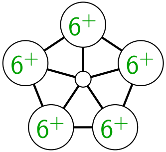


$\frac{1}{5} - 2(\frac{1}{10})$

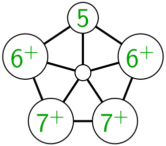


$2(\frac{1}{5}) - 4(\frac{1}{10})$

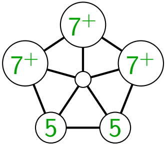
$d(v) = 5$: Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$



$2(\frac{1}{3}) + 2(\frac{1}{5})$



$3(\frac{1}{3})$

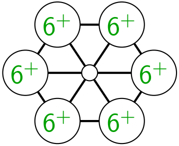
All Vertices are Happy (continued)

Discharging Rules

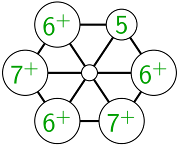
(R1) Each 6-vertex gives each 5-nbr $\frac{1}{5}$.

(R2) Each 7^+-vertex gives each 6-nbr $\frac{1}{10}$ and each 5-nbr $\frac{1}{3}$.

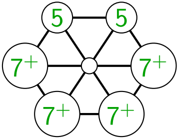
$d(v) = 6$: Can give net charge $6 - 6 = 0$.



0

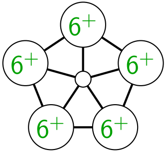


$\frac{1}{5} - 2(\frac{1}{10})$

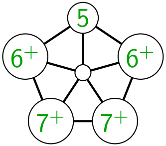


$2(\frac{1}{5}) - 4(\frac{1}{10})$

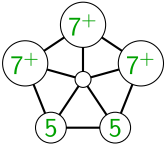
$d(v) = 5$: Must get net charge $6 - 5 = 1$.



$5(\frac{1}{5})$



$2(\frac{1}{3}) + 2(\frac{1}{5})$



$3(\frac{1}{3})$

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- Thanks to R. Thomas and UIUC math for pictures in intro!